

# NOTES FOR MAA MINICOURSE

## TEACHING A COURSE IN THE HISTORY OF MATHEMATICS

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For more information on the course, see Fred Rickey's webpage:  
<http://www.dean.usma.edu/math/people/rickey/hm/mini/default.html>

## Who Will Your Audience Be?

In planning any history of mathematics course, there is no more important question than who your audience will be. If you misjudge the answer to this question, your course will be a disaster. Your students will be very unhappy. You will be distraught.

You should be able to determine your audience in advance. If the course is already on the books, then the syllabus will list some prerequisite. Perhaps the course is even required of some group of students. Knowing your audience settles the hardest question.

If you are teaching a course that has never been taught at your school, at least not in recent years, then you will have to drum up an audience. You will know some of these students and can safely assume that the friends they persuade to sign up too are somewhat like them.

Here are some questions that you should think carefully about:

- What level are your students? Freshman, Junior-Senior, Graduate?
- How good are your students? Are you at a community college, a selective liberal arts college, an open enrollment state university, or a graduate university?
- How much mathematics do they know? Calculus? Several abstract mathematics classes? Is the course for liberal arts students?
- What will they do after graduation? Will they become elementary teachers, secondary teachers, graduate students, or take a job? Is this a course for poets?
- If your students are prospective teachers, what history will benefit them?
- Why are the students taking the course? Is it required of their major? Why is it required? Is it an elective class? Are they taking it because you will teach it? Are they seriously interested in history?
- What do students need to get out of the course? How much “fact” do they need to know?
- Is this a capstone course for mathematics majors that is intended to tie together what they have learned in other courses?

No matter who your audience is, you need to be aware that they will not know enough mathematics and they will not know enough history (and neither will you). You have to teach history; you have to teach mathematics. This provides a considerable challenge.

A common audience is prospective secondary teachers. Their future students will definitely benefit if they know something of the history of geometry, trigonometry, and algebra. Should you do something in the history course to review and expand their knowledge of this mathematics (where else in the standard undergraduate curriculum has their knowledge of these fields been expanded)? Certainly the prospective secondary teacher needs to know something of the history of quadratic equations. Why are they called “quadratic” equations? Doesn’t “quad” mean four? They may never have the need to solve a cubic equation, but learning how and learning something of their history will make it clear that complex numbers arose from solving cubic equations (everyone knows that  $x^2 = -1$  has no solutions; why would mathematicians want to make up solutions for this equation?). If you are teaching prospective teachers, you probably need to stick mostly to the history of topics that are in the high school curriculum. You need to

give these students experience with using history in creating lessons in school. They need to understand why certain topics were developed and what their original and later uses were. And you should be familiar with the *Standards* of the NCTM, because the students will be. Thus, you should try to emphasize discovery techniques in class, of course from a historical point of view, so that you model the teaching your audience will be doing later in school.

If the class will consist of mathematics majors who intend to go to graduate school, then perhaps the best thing for them is to understand how modern abstract mathematics arose. A discussion of non-Euclidean geometry would benefit them (isn't it interesting that this is the only undergraduate course that is customarily taught with a large historical component; the reason is clear – the only motivation for the material is historical). How has the creation of non-Euclidean geometry changed the way we think about our world? Contrast the Declaration of Independence with the Gettysburg Address; think about how the geometry of the fourth dimension has influenced abstract art – aren't these themes also important for the liberal arts student? So too would knowledge of the rise of abstract algebra benefit the prospective graduate student. Finally, an understanding of why the rigorization of mathematics took place in the nineteenth century is very important for a new graduate student (for their teachers in graduate school almost certainly won't – should I say can't – explain). So if your audience is prospective graduate students, perhaps a topics course in nineteenth century mathematics is what they need.

If your class is for poets, what do you want them to take away from it? Perhaps the Copernican revolution should be part of the course, for it certainly changed the way we understand the world. Shouldn't they understand how statistics influences what we do today? This calls for a substantially different approach. But these students should also learn that although many mathematical ideas originated in response to real problems in a particular civilization, nevertheless, there have always been “mathematicians,” people who took the basic mathematical ideas and then developed them far past the original question. These people were excited by mathematics and did not worry about whether their ideas had any practical use. Curiously, it has turned out in many instances that these “abstract” mathematical ideas eventually found practical application.

Should you be lucky enough to have a group of mathematically knowledgeable students who have a serious interest in history, then perhaps you want to include the careful reading of original sources. This approach will give your students a deep appreciation of what historians do, so perhaps this should be one of your aims in any course you teach.

Finally, the students may be taking the course because you are teaching it. They may just want to know what interests you. This allows considerable leeway in the design of your course.

## **What are the Aims of Your Course?**

What are your goals in your history of mathematics class? This is, of course, an idiosyncratic question, but I do think that you should have specific goals and that you should state them explicitly to the students. Here are Rickey's aims, as stated in his syllabus:

1. To give life to your knowledge of mathematics.
2. To provide an overview of mathematics---so you can see how your various courses fit together and to see where they come from.
3. To teach you how to use the library and internet (important tools for life).
4. To show you that mathematics is part of our culture.
5. To indicate how you might use the history of mathematics in your future teaching.
6. To improve your written communication skills in a technical setting.

Katz has some additional aims:

1. To show that mathematics has been developed in virtually every literate civilization in history, as well as in some non-literate societies.
2. To compare and contrast the approaches to particular mathematical ideas among various civilizations.
3. To demonstrate that mathematics is a living field of study and that new mathematics is constantly being created.

Now you may well have different goals, but I do encourage you to think about this in advance and to explicitly state some goals to your students. This will help you in designing the course and in justifying it to your colleagues.

## **Counteract negative views in your department**

Among some mathematicians the history of mathematics is not regarded as a serious pursuit. They believe it is something that people do when they can no longer do research in mathematics. You may encounter this view in your department, or it may be present but your colleagues don't express it.

It is worth your while to spend some time talking to your colleagues about your course. Point out to them that you are doing significant amounts of mathematics in your course (give some illustrations). Point out that it is not a course in anecdotes. Yes, you do relate this information, but you are really involved in a serious intellectual pursuit. You have high demands on the work that the students do. They must master a great deal of material and they are required to write about mathematics in a way that shows that they have mastered the details.

## **Aims of other people**

If you examine various syllabi, you will find a great number of aims in teaching history of mathematics courses. Here are some of them:

- To explain the use of historical material in the classroom.
- Considerable emphasis on how to find out information and how to present it.
- Get students reading original sources.
- Critical analysis of factual materials.
- To deal with methodological and philosophical controversies.
- The art of communication.
- To impart the view of mathematics as a continually developing human activity.
- To understand different ideas of mathematics held at different times.
- To give students a chance to reflect on their chosen field.
- To get you interested in a particular mathematical topic.
- To look at some of the great unsolved problems of mathematics.
- To examine in detail some aspects of modern mathematics.
- This course is a supplement to a course on Galois Theory.
- To see the strands of mathematics which fused into modern statistics.
- To enable students to widen, deepen and transform their views of the nature of mathematics, keeping in mind their future teacher objectives.

You should look at other peoples' syllabi and borrow their good ideas.

## Types of History of Mathematics Courses

There are many types of history courses that you can design, including:

- Survey
- Theme
- Topics
- Sources
- Readings
- Seminar

In typical mathematical fashion, we begin with definitions.

A **survey** course is one that discusses a broad range of the history of mathematics including a variety of topics over many consecutive time periods. Most courses taught to undergraduates are of this type. Still, depending on the audience, the topics selected will vary. For example, the following survey courses will be quite different from each other:

- History of mathematics for prospective elementary teachers.
- Mathematical ideas leading to the calculus.

A **theme** course selects one mathematical topic to concentrate on for the entire term.

Some examples:

- The History of the Calculus.
- Women in Mathematics.
- Non-Euclidean geometry.
- The mathematics of Isaac Newton.

A **topics** course chooses several mathematical topics to discuss and bases the whole course on ideas related to these topics. Here are a few examples:

- Great Theorems of Mathematics.
- Hilbert's problems.

A **sources** course is based on the reading of original sources, usually in English translation. One good example of such a course is taught by Gary Stoudt:

[http://nsm1.nsm.iup.edu/gsstoudt/history/ma350/sources\\_home.html](http://nsm1.nsm.iup.edu/gsstoudt/history/ma350/sources_home.html)

A **readings** course is organized around reading a variety of types of material. Examples:

- Mathematics in Russia.
- Seminal papers in and about topology.
- Important recent papers in the history of mathematics.

A **seminar** course is organized so that there is more discussion than lecture and so that several individuals take the lead in developing the individual topics chosen.

We have begun with definitions, but you will note that these are not as clear as a mathematician might wish, and they are certainly not exclusive. For example, several people have used *Journey through Genius* by Bill Dunham as a textbook. This combines some of the idea of a topics course with a sources course.

## Textbooks for a Survey Course in History of Mathematics

This list of possible textbooks may be helpful to teachers when choosing a textbook for a survey course in the history of mathematics. There are many other textbooks that have been used for other styles of history of mathematics courses, but attention is restricted here to the textbooks that are currently used in most mathematics history survey courses taught in North America:

Carl Boyer, *A History of Mathematics*

David M. Burton, *The History of Mathematics: An Introduction*

Howard Eves, *An Introduction to the History of Mathematics*

Victor J. Katz, *A History of Mathematics: An Introduction*

Victor J. Katz, *A History of Mathematics, Brief Edition*

Ronald Calinger, *A Contextual History of Mathematics*

William Berlinghoff and Fernando Gouvêa, *Math Through the Ages: A Gentle History for Teachers and Others*

Roger Cooke, *The History of Mathematics: A Brief Course*

Jeff Suzuki, *A History of Mathematics*

Excerpts from reviews of each of these texts are given below (no attempt has been made to list every review of these texts). We have paid special attention to the comments in the reviews that state opinions about what is most desirable in a history of mathematics textbook, as well as what is noteworthy and lacking in each of these. However, no attempt has been made to summarize the contents or to repeat information that appears in several of the reviews. You are encouraged to look up these reviews and to read them in detail.

**Carl B. Boyer, *A History of Mathematics*, New York: John Wiley, second edition, 1989. ISBN 0-471-09763-2.**

Reviewed by Paul J. Campbell, *Mathematics Magazine*, 62 (1989), 284.

This five line review indicates that the first 22 chapters have been left virtually unchanged while those dealing with 19th and 20th century mathematics have been revised and expanded. References and bibliographies have been updated.

Review of the first edition (1968) by Phillip S. Jones, *American Mathematical Monthly*, 76 (1969), 1163-1165.

“In a historical treatise one looks for scholarship in marshaling the facts, for insight and a broad view in portraying the changing trends, themes, and interrelationships both within mathematics and between mathematics and other aspects of culture. One may also be concerned for the coverage and for the quality of the evaluation of forces directing progress. This book scores high in all of these respects.” Recent historical scholarship is taken into account and the bibliographies are “very helpful to students doing individual projects.” Jones is concerned that the student will not find this book easy reading for it is “packed with condensed information and interpretation.” The student should fill in the details. “The book is remarkably free of errors.” “In conclusion, the book rates an excellent score as both a scholarly product which sets a good example for young scholars and as a textbook in an undergraduate course.”

**David M. Burton, *The History of Mathematics: An Introduction*, Boston: McGraw Hill, sixth edition, 2007, xii + 788pp. ISBN 0-07-305189-5.**

First edition (1985) reviewed by Ronald Calinger, *ISIS*, 81 (1990), 545-546.

This solid survey of the history of mathematics is written in a lively style and contains “a rather idiosyncratic store of information.” “China, the Hindus, and medieval Islam are largely overlooked,” as is Napier's work on logarithms. [This has changed in more recent editions.] “The interspersed treatment of the development of the calculus is not adequate.” [After reading the sections of Boyer and Katz on the Calculus you should judge for yourself if this treatment is deficient; later editions of Burton are better.]

Reviewed by David Wheeler, *The College Mathematics Journal*, 17 (1986), 373-375.

“This is a history book in a familiar mold. . . . it pictures mathematics . . . as progressively and inexorably unfolding, brilliantly impelled along its course by a few major characters, becoming the massive edifice of our present inheritance.” It is based on up to date secondary sources. The author leaves out some topics because of their technical difficulty, but should there be no explanation of the Eudoxan theory of proportion, and is the Cantor-Dedekind theory of irrationals “a story best avoided” (p. 577)? “I do not think anyone could find from this book how and when the concept of function surfaced, and how it developed subsequently.” Nonetheless, many topics are treated satisfactorily, including Greek geometry and non-Euclidean geometry. My chief problem with the book, though, is one that Eves’ book sets me, too. They both offer a large number of historical facts and anecdotes without making any real attempt to show the reader how a sense of history can deepen understanding of mathematics [Instructors should think hard about how they can do this in their courses], shed light on the roots of mathematical ideas, give a grasp of the role that mathematics plays in human society, and so on. But for what other purposes would one be writing a history of mathematics for undergraduates? Many periodical references accessible to undergraduates are given. But neither Burton nor Eves “is really any help to a reader who wants to know what to read next on a particular topic or period.” “Unfortunately, the index, though long, has not been prepared with sufficient care.” Examples are given in the review.



**Howard Eves, *An Introduction to the History of Mathematics*, Saunders Publishing Co., sixth edition, 1990, xviii + 775 pp. ISBN 0-03-029558-0.**

Review of the first edition (1953) by Phillip S. Jones, *The American Mathematical Monthly*, 61 (1954), 433-436.

There has long been a need for a book on the history of mathematics which was suitable as a text with undergraduates at the junior-senior level, . . . Eves' book was written expressly for this purpose. It succeeds remarkably well . . . A unique and usable feature of the book is the set of "Problem Studies" at the end of each chapter. . . . Although the reviewer would prefer more emphasis on a topical approach . . . the book is not a mere recital of names and dates, but does well in an attempt to stress the growth of ideas and interrelationships between them for readers who are not too advanced or mature mathematically. However, the first edition has "a few places where condensed discussion, the use of modern symbolism, or the author's viewpoint produce what from the reviewer's viewpoint are slight distortions of history." Several examples are given; I did not check that they were corrected in later editions, but would presume that they have been. Historians will be amused to learn that the cost was \$6.00.

Review of the second edition (1964) by Phillip S. Jones, *Scripta Mathematica*, 28 (1967), 168.

A readable book but it "lacks pictures and reproductions of portions of original works which the reviewer would like to see."

**Victor J. Katz, *A History of Mathematics: An Introduction*, 3<sup>rd</sup> ed. Boston: Addison-Wesley, 2009, xvi + 976pp. ISBN 0-321-38700-7**

Review of first edition by Tom Archibald, *Historia Mathematica*, 23 (1996), 89-92.

What we want in a history of mathematics textbook "is a book which is accessible to students who have mastered very little beyond the calculus, which is mathematically respectable, historically very solid, which can serve as a source of problems and projects for the teacher to assign, which can serve as a useful reference, and which can guide the student and the instructor in their further study. Katz's book succeeds admirably on all these counts, and many more besides." (p. 90) "The combination of clear exposition, superior documentation, and mathematical richness should be actively sought by all who teach such courses." (p. 92) Each chapter ends with references that are of the highest quality available, including recent journal literature. "The book treats mathematics outside Europe seriously and at length." (p. 91). Archibald notes that he has used this book successfully with junior level math majors, but warns that it would be too much for weaker students [This is definitely true.], and would have to be supplemented for more sophisticated students.

Review of first edition by Judith Grabiner, *The Mathematical Intelligencer*, 16:4 (1994), 73-76.

The full-color cover with its detail of The Ambassadors of Hans Holbein the Younger prepares us "for an attractive, well-illustrated book that respects original sources and presents material

from them, and that stresses the cultural setting of mathematics.” While Katz covers the standard topics, the outstanding features of this book is that he closely follows original sources and has an “excellent treatment of mathematics outside the European tradition.” Grabiner has used this book in class and was pleased with the outcome. In a semester one can do the first 12 chapters, up through the calculus of Newton and Leibniz. The index gives phonetic pronunciation for non-English names, “a feature my students found surprisingly empowering.” “The most serious criticism one can make is that Katz's coverage reflects the limitations of twentieth-century scholarship. The book “is not itself one of path-breaking scholarship. . . . Much remains to be studied.” Yet, Katz has “produced an excellent and readable text, based on sound scholarship and attractively presented.”

Review of first edition by David E. Rowe, *ISIS*, 85 (1994), 125.

Many history of mathematics courses are taught by “mathematicians with only scant knowledge of history” using “textbooks that betray little knowledge of either primary sources or the scholarly literature concerning them, treat mathematical ideas and problems in Platonic fashion as enjoying an existence largely independent of their cultural content, confine themselves to those aspects of (mostly pure) mathematics that have found a secure place in modern mathematical textbooks, and display a marked Eurocentrism that pays but little attention to the role of mathematics in non-Western cultures.” Katz “manages to avoid all of these standard pitfalls.” This book is a “highly readable, comprehensive, and exceptionally well-researched work” that “successfully bridges the yawning gap that until now has separated scholarship from teaching in the field of history of mathematics.” It is designed for use by prospective secondary teachers and “sets a new standard for historical writing in this genre.”

Review of first edition by James J. Tattersall, *The College Mathematics Journal*, 25 (1994), 347-348.

“What really distinguishes this book from any other history of mathematics text is undoubtedly its exercises.” (p. 348). They are carefully chosen and based closely on the original texts. The student who does them diligently will become acquainted with many important texts, will come to understand the cultural settings of the text, and will acquire a greater understanding of mathematics. Some exercises in each chapter involve ways of bringing history into the ordinary mathematics classroom. But the problems are hard for most students. They must read the text carefully to learn how to do the problems and also must review much of their mathematical knowledge. Tattersall's course was for MAT students. They met once a week for two hours for a semester and covered most of the first 13 chapters (through the calculus and Non-Euclidean Geometry). “At the end of the semester, all students agreed that they had learned a lot of mathematics. They felt that Katz’s text not only had helped make them better teachers of mathematics but had given them a greater appreciation for the subject and more insight into it.” (p. 348). The students would have liked “time lines using line intervals” as well as more maps. There is plenty of material here for two semesters.

Review of third edition by Jacqueline Stedall, *MAA Reviews*, 2008

“Victor Katz’s *A History of Mathematics: An Introduction* is already well known as a comprehensive textbook in the history of mathematics, courageously covering material from

‘ancient civilizations’ to ‘computers and their applications’ in just under 900 pages. In content the third edition (2009) remains essentially the same as the second edition (1998) but it has also been revised and updated. What is new?

Most significantly, since working on the previous edition Katz has edited *The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook*, and this has led to substantial changes to *A History of Mathematics*, with new sections on each of those five regions. Thus new scholarship on the ancient and medieval world has been rapidly introduced into a widely used textbook. In the better known field of Greek mathematics, Euclid’s *Elements*, so essential to an understanding of later European mathematics, now benefits from a longer treatment than before, and there is also new discussion of the Archimedes palimpsest and the discoveries that have arisen from it.

For later periods of history, where the focus moves to western Europe, there is are helpful separations of material so that, for example, Viète and Stevin, who were so very different in motivation and output, now each have their own subsections; similarly, the mature calculus of Newton and Leibniz is treated separately from the earlier seventeenth-century ‘beginnings of calculus’.

Katz’s revised discussion of eighteenth-century calculus is also differently arranged, with a new section on translating the methods of Newton’s *Principia* into differential calculus. For the eighteenth century, probability, algebra, and geometry now have a whole chapter each, and there is also a new chapter on probability in the nineteenth century, so that it is now possible to follow the distinct threads of algebra, analysis, probability, and geometry through the second half of the book. The volume ends with a new discussion of twentieth-century solutions to some old problems: Fermat’s last theorem, the four-colour problem, and the Poincaré conjecture.

Despite the welcome updating, some of the problems of earlier editions remain. One is the translation of historical mathematics into modern notation. Of course this is a useful, sometimes necessary, thing to do to aid understanding, but to do it without ever returning to the original texts obscures historical reality. Katz’s account of Newton’s discovery of the general binomial theorem, for example (pp. 547–548), claims that Newton did so by means of an elegant modern formula. This bears little relation to the manuscript evidence of Newton pursuing a lengthy process of trial and error and empirical observation, with a formula of sorts emerging only at the end.”

**Victor J. Katz, *A History of Mathematics: Brief Edition*, Boston: Addison-Wesley, xv + 560 pp. ISBN 0-321-16193-9.**

Review by Robert McGee, *Convergence*, 2005

“Victor Katz’s goal was to write a book which ‘concentrates on the history of those topics typically covered in an undergraduate curriculum or in an elementary or high school.’ Although difficult choices had to be made in downsizing his excellent book, *A History of Mathematics: An Introduction*, the result is a very fine book that serves its audience well. The first chapter examines the contributions of the Egyptians and the Babylonians. Each culture is studied separately, allowing an opportunity to compare and contrast their accomplishments. Chinese and Indian mathematics each receive a separate chapter in this book. Together with the chapter on the mathematics of the Islamic world, these three chapters provide a very strong introduction to non-

western mathematics. Three chapters are devoted to Greek mathematics and another three chapters to those topics in high school mathematics that might be classified as precalculus mathematics. Chapter 11 introduces the calculus of the seventeenth century. The remaining nine chapters of the text carry the history of mathematics into the twentieth century. Most of this material would be more appropriate for those who plan to teach on the college level, although selected topics could be incorporated into a course for pre-college teachers. I integrate problem solving into my course through the use of historical problems. This book contains a large and varied collection of problems. Answers are given to selected problems. The author also includes a short appendix to facilitate the use of the textbook for teachers of mathematics including a list of topics and the relevant sections of the text where they are covered.”

**Ronald Calinger, *A Contextual History of Mathematics*, Upper Saddle River, NJ.: Prentice-Hall, 1999, xxviii + 758 pp. ISBN 0-02-318285-7.**

Review by Eisso Atzema, *MAA Reviews*, 2000.

“In my experience, and also in that of others with whom I have discussed this, it is almost impossible to get beyond the history of the early calculus in such a course. In that light, it is refreshing that Calinger's *A Contextual History of Mathematics* does not cover any mathematics beyond the early 18th century at all. This does not mean that the volume is slim, but at least it does not contain material which you can tell in advance that you will never cover. Clearly, this is a book written by somebody with ample experience teaching history of mathematics. A concern for delivery also shows in the rich collection of anecdotal stories the text is peppered with.” ... “In his long introduction, Calinger explains how he feels his book is different from other comparable textbooks. His claim is that his book integrates the development of mathematics with the development of society at large more than other textbooks in the field. And indeed, Calinger shows himself to be sensitive to the societal context in which mathematics develops and he probably reserves more space for discussion on political and cultural events than authors such as Katz and Cooke do. In the case of Italian Renaissance Algebra, for instance, Katz gets to the algebra almost right-away. Calinger, in contrast, needs several pages of introduction in which he manages to put a lot of art history as well as political history. Certainly, all of that information serves some purpose, but much of it strikes me as ornamental more than anything else — at least in an introductory textbook. If the directness of most textbooks in history of mathematics is reminiscent of the austerity of Roman architecture, Calinger's work definitely is baroque in style.”... “The mathematics itself is presented competently and there are no surprises as to the choice of material. Also, the book contains more than enough concrete examples of the mathematics under discussion to allow for a course with a heavy mathematical component — an important issue, since most history of mathematics courses are taught within mathematics departments and some mathematical content is expected. I frankly do not see anything that makes Calinger's book very different from what is already on the market. I certainly could teach from the book, although other books would have my preference.”

Review by John McCleary, *Choice*, 1999.

“The author's goal in this work is a general history of mathematics from prehistory through its premodern developments, to be read by students, mathematicians, and historians of science. As

the title indicates, the intention is to describe, in considerable detail, the context in which mathematics developed. In a lengthy prologue, the author discusses historiographic methods and it is clear that the author has considered the scholarship on the history of early mathematics in depth. However, the results of his far-ranging reading do not measure up to his intentions. He writes that the book must remain provisional in its conclusions. This is the major weakness of the work. ... The illustrations in this book appear largely unattributed and sometimes randomly. This does not achieve the level of scholarship that naïve readers of the book ought to see. Furthermore, there are errors in some of the mathematics that does appear.”

**William Berlinghoff and Fernando Gouvêa, *Math through the Ages: A Gentle History for Teachers and Others*, Expanded Edition, Washington: MAA, 2004, 288pp. ISBN 0-88385-736-7.**

Review of first edition by Victor Katz, *MAA Reviews*, 2002

“Where does  $\pi$  come from? Why should we be interested in negative numbers, or square roots of negative numbers? How did people ever figure out the quadratic formula? The answers to these and many other similar questions asked by college and secondary teachers and their students can be found, in easily accessible form, in this wonderful little book by two faculty members at Colby College. Although knowledge of the history of mathematics is an important tool for mathematics teachers at the secondary and college level, it is not always simple to find out about the history of a particular topic. There are many solid texts in the field, but often the history of the quadratic equation, for example, is spread among several chapters because it extends through many cultures and many centuries. In this book, on the other hand, the basic story can be found immediately by turning to chapter 10. The authors call this book ‘a gentle history,’ and indeed it is. It very carefully gives the reader capsule histories of such topics as zero, symbols in algebra, solving cubic equations, the Pythagorean theorem, sines and cosines, and elementary statistics, among many others. Each short chapter discusses the history of a particular idea, often spanning centuries and civilizations, but always clearly and concisely. It is written so that a teacher can dip in wherever she wants, find the information she needs, and use the information in classes or answer questions of her students. But if the teacher wants more than these capsule histories, the first section of the book, entitled ‘The History of Mathematics in a Large Nutshell,’ will help. Here, the authors treat the history of mathematics chronologically, from its beginnings in ancient Egypt and Mesopotamia up to the development of computers in the late twentieth century. This section thus provides a broad overview that will help the reader place the individual chapters in context. And if the reader wants to go deeper into any particular subject, or into the history of mathematics as a whole, the authors again provide guidance, first with a suggested reference shelf of books to which to turn to find more information, then with a list of ‘fifteen historical books you ought to read’ (in their entirety), and finally with a brief tour of historical sources on the Internet. Finally, the book contains a marvelous bibliography of other works referred to in the various chapters.”

Review of second edition by Marion Cohen, *MAA Reviews*, 2008

“The expanded edition came out in 2004, and includes problems and projects at the end of each of the short sketches (and only projects at the end of the large nutshell). The problems are very

well chosen — not too easy, not too difficult, and helpfully worded. For example, p. 243 (in that edition), on set theory and infinities: “How much less than 1 is each term in this [convergent] sequence...” and “Even a toddler can see if she got her fair share of gum drops by matching them up with her brother’s share.” Also, the last problem in most of the sections strives to give readers a sense of the history of it all (“that timeline feeling”). For example, p. 244: “Georg Cantor lived and worked during a time of political and social turmoil in central Europe, much of it involving Germany. (a) What was the foremost political event in Germany in 1871... (b) What famous German composer...?” Etc. This device not only gives “that timeline feeling”, but also connects math history events with *other* history events. Thus readers learn history, period, not only math history.

As for the projects, pretty much ditto. For example, continuing with p. 244, the first project, “Write an essay comparing and contrasting the mathematical concept of infinity with the idea of infinity as it occurs in some other subject area...” That’s just plain fun! When I teach any math subject, history or actual math, I always try to emphasize how math terms are so often derived from “regular life terms”, so that students can appreciate the appropriateness, and in some cases the poetic-ness, of the math terms. The second project (proving things like “the set of rational numbers is the same size as the set of natural numbers”) is fun *mathematically*, and a bit challenging for those who don’t know Cantor’s work. And the third project, like the third problem, helps give readers a sense of history, and perspective, mathematically and otherwise.

I do have second thoughts about the phenomenon of problems and projects in textbooks. (Keep in mind that this is “just me...”.) I realize that most textbook publishing companies want them, and so do most educational establishments. But I speculate that there might be a tradeoff to the advantages of the phenomenon. *Without* the problems and projects, our class did just fine. As described earlier in this review, we explored, we conversed, we played math games, and through these mostly spontaneous activities some of the very problems and projects from the new edition came up. Of course, there were problems and projects that the new edition contains that we did not think of, and also vice versa. But the process was more natural the way we did it, using the old edition, and if I were to teach the course again, I’d want to continue in this vein.

I think that, for some students, the presence of problems and/or projects (even if they’re not specifically assigned in the course) serves as reminder that this is, indeed, a *textbook* — something which students (in particular, math-anxious students) often have negative associations about. Thus, for some, there could be something a little (or a lot) intimidating, perhaps joy-squelching, about problems and projects presented so “officially”, “in writing”. (As we know, I did assign problems and projects, or the students chose them, but they weren’t so ubiquitous.)”

**Roger Cooke, *The History of Mathematics: A Brief Course*, second edition. New York: John Wiley & Sons, 2005, xvii + 607pp. ISBN 0-471-442159-6**

Review by Amy Shell-Gellasch, *MAA Reviews*, 2007

“The second edition of Cooke’s *The History of Mathematics: A Brief Course* is a jewel. It is notable for what it includes as well as what it does not. But most importantly, it is a jewel for its presentation.

The majority of history of mathematics texts are ordered chronologically. And to be honest, it is nearly if not completely impossible to present an historical topic without some chronology. But this text is organized first by theme. The text is made up of seven parts broken into chapters. The first part consists of chapters on the origin and prehistory of mathematics, mathematical cultures, and women in mathematics. Part two is on numbers, from counting to combinatorics. Part three is on space up through point-set topology. Parts on algebra, analysis and finally mathematical inferences follow, as well as a section of beautiful color images. Given this organizational structure, several themes, cultures, and people make numerous appearances in different sections. I found this approach refreshing as well as informative.

Of the items included that I found interesting were discussions of Japanese, Australian, New Zealand and Mexican mathematics. These cultures are rarely or only briefly mentioned in typical history texts. Cooke takes pains to explain some of the dead ends taken by mathematicians as well as how the process of mathematics from an earlier era is different than current practices. At times Cooke goes into great detail in order to show the reader the thought process used by the individual in question. He gives information on those individuals who made concurrent or prior discoveries along with information on the individuals traditionally given credit for a discovery. Information about original sources is included in places as well. The references are a combination of new research and old classics now often ignored. These attributes are not commonly found in mathematical history texts and are what make this volume stand out.

Developed as a text for his undergraduate introductory math classes, Cooke (mathematics and statistics, U. of Vermont) tours the origins of the field in Western and other traditions ((Hindu, Chinese, Korean and Japanese, and Islamic), and its modern evolution from the Middle Ages to the present. Equally intriguing is what the author notes he decided to edit out: e.g. nearly all biography. Chapters feature problems and commentary on why each problem was important, how it was solved, and implications of the solution for further developments. Concluding chapters link mathematics to contemporary US scientific policy and social issues (math in Nazi Germany and the Soviet Union; women and minorities in the field). Written by one of the foremost experts in the field, *The History of Mathematics: A Brief Course* is substantially revised in the second edition. This acclaimed text-now reorganized topically rather than geographically-begins with first applications of counting and numbers in the ancient world, and continues with discussions of geometry, algebra, analysis, probability, logic, and more. Discussions of women in the history of mathematics make this a very thorough, inclusive resource....This book is a fascinating look at the history of mathematics, and is sure to inspire even the most devout haters of numbers.”

**Jeff Suzuki, *A History of Mathematics*. Upper Saddle River, NJ: Prentice Hall, 2002, xiii + 815pp. ISBN 0-13-019074-8.**

Review from Amazon (2005)

“This history of mathematics is a mathematics text, allowing students to understand the history of mathematics by using the tools mathematicians of the past used. Emphasis is on numeration, computation, and notation, mathematical results in their original form with their original arguments, and mathematics as an evolving science. Although the book can be used selectively

by students with any level of background, it is best if students have had a previous course in calculus. Suzuki teaches at Boston University [now at Brooklyn College].”

A new textbook that has recently been published in England is

**Luke Hodgkin, *A History of Mathematics: From Mesopotamia to Modernity*. Oxford: Oxford University Press, 2005.**

Although the style is quite different from the books noted above, one could certainly also consider this book as a text for a survey course.

This is an excerpt of a review by Victor Katz in *Math Reviews*, 2006:

“This new history of mathematics text is different from the general run of such texts, in that its emphasis is on the questions historians ask, rather than on the answers. It attempts to have students think about historical events, consider various alternate explanations, and come to their own conclusions, rather than trying always to give the author's opinion of the history. Recognizing that universities are more and more frequently requiring a “historical component” in their mathematics curriculum, in order “to produce a less limited and so more marketable science graduate,” Hodgkin has attempted to meet this need, not by just reciting the “facts” but by attempting to produce in the students a critical eye toward history.

Hodgkin's text is very limited in scope, compared to many texts in the history of mathematics. Each of his ten chapters considers a fairly narrow topic, beginning with setting the stage through raising major questions, then following with a brief survey of the sources for the period, and finally discussing in some detail the relevant “history.”

.....

The book is well-written, with a typically British dry humor. Although there are a few errors that may well confuse readers, including the use of reprint dates of important works rather than the original dates, in general it is accurate and up-to-date. But whether such a book will be useful in the typical history of mathematics course taught in the United States is a question. Hodgkin constantly refers to the difficulties in a student gaining access to what literature exists on various subjects, but then he does not make too much of an effort to replace that access with his own discussion. And because the choice of topics is very limited and rather idiosyncratic to begin with, the typical university mathematics major will not gain much insight into the history of many important ideas. Nevertheless, the book constantly reiterates the major point that history is not just a “collection of facts,” but that it involves detailed questioning of the primary and secondary sources that exist and requires students to search for and justify their own answers to important questions. I would hope that many teachers, at least, will use the book to inspire their own courses, even if they cannot use the book itself as a text.”



## **Reading List on History of Special Topics in Mathematics**

### **Ancient Mathematics**

Otto Neugebauer, **The Exact Sciences in Antiquity** (Princeton: Princeton University Press, 1951)

B. L. van der Waerden, **Science Awakening I** (New York: Oxford University Press, 1961)

B. L. van der Waerden, **Geometry and Algebra in Ancient Civilizations** (New York: Springer, 1983)

Jens Høyrup, **Lengths, Widths, Surfaces: A Portrait of Old Babylonian Algebra and Its Kin** (New York: Springer, 2002)

Jöran Friberg, **A Remarkable Collection of Babylonian Mathematical Texts** (New York: Springer, 2007)

Eleanor Robson, **Mathematics in Ancient Iraq: A Social History** (Princeton: Princeton University Press, 2008)

Richard J. Gillings, **Mathematics in the Time of the Pharaohs** (Cambridge: MIT Press, 1972)

Shen Kangshen, John S. Crossley and Anthony W.-C. Lun, **The Nine Chapters on the Mathematical Art: Companion and Commentary** (Oxford: Oxford University Press, 1999)

Denise Schmandt-Besserat, **Before Writing: From Counting to Cuneiform** (Austin: University of Texas Press, 1992)

### **Greek Mathematics**

Thomas Heath, **A History of Greek Mathematics** (New York: Dover, 1981) (reprint of 1921 original)

Wilbur Knorr, **The Evolution of the Euclidean Elements** (Dordrecht: Reidel, 1975)

Wilbur Knorr, **The Ancient Tradition of Geometric Problems** (New York: Dover, 1993) (reprint of 1986 original)

D. H. Fowler, **The Mathematics of Plato's Academy: A New Reconstruction** (Oxford: Clarendon Press, 1987, 1999)

J. Klein, **Greek Mathematical Thought and the Origin of Algebra** (Cambridge: MIT Press, 1968)

E. J. Dijksterhuis, **Archimedes** (Princeton: Princeton University Press, 1987)

S. Cuomo, **Ancient Mathematics** (London: Routledge, 2001)

Benno Artmann, **Euclid: The Creation of Mathematics** (New York: Springer, 1999)

Michael N. Fried and Sabetai Unguru, **Apollonius of Perga's *Conica*: Text, Context, Subtext** (Leiden, Brill, 2001)

Reviel Netz, **The Shaping of Deduction in Greek Mathematics: A Study in Cognitive History** (Cambridge: Cambridge University Press, 1999)

Reviel Netz, **The Transformation of Mathematics in the Early Mediterranean World: From Problems to Equations** (Cambridge: Cambridge University Press, 2004)

Reviel Netz, **The Works of Archimedes: Translation and Commentary**, Volume 1 (Cambridge; Cambridge University Press, 2004) (Two more volumes are forthcoming.)

Christian Marinus Taisbak, **Euclid's Data: The Importance of Being Given** (Copenhagen: Museum Tusculanum Press, 2003)

S. Cuomo, **Pappus of Alexandria and the Mathematics of Late Antiquity** (Cambridge: Cambridge University Press, 2007).

## **Medieval Mathematics**

Ulrich Libbrecht, **Chinese Mathematics in the Thirteenth Century: The Shu-shu chiu-chang of Ch'in Chiu-shao** (Cambridge: MIT Press, 1973)

Li Yan and Du Shiran, **Chinese Mathematics: A Concise History**, translated by John N. Crossley and Anthony W. C. Lun (Oxford: Clarendon Press, 1987)

Jean-Claude Martzloff, **A History of Chinese Mathematics**, translated by Stephen S. Wilson (Berlin: Springer, 1997)

J. Lennart Berggren, **Episodes in the Mathematics of Medieval Islam** (New York: Springer, 1986)

George Gheverghese Joseph, **The Crest of the Peacock: Non-European Roots of Mathematics** (Princeton: Princeton University Press, 2000)

Edward Grant and John E. Murdoch, eds., **Mathematics and Its Applications to Science and Natural Philosophy in the Middle Ages: Essays in honor of Marshall Clagett** (Cambridge: Cambridge University Press, 1987)

David E. Lindberg, ed., **Science in the Middle Ages** (Chicago: University of Chicago Press, 1978)

Roshdi Rashed, **The Development of Arabic Mathematics: Between Arithmetic and Algebra**, translated by A. F. W. Armstrong (Dordrecht: Kluwer Academic Publishers, 1994)

George Saliba, **Islamic Science and the Making of the European Renaissance**, (Cambridge: MIT Press, 2007)

Victor J. Katz, editor, **The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook** (Princeton: Princeton University Press, 2007)

B. Datta and A. N. Singh, **History of Hindu Mathematics** (Bombay: Asia Publishing House, 1961) (reprint of 1935-38 original)

Kim Plofker, **Mathematics in India: 500 BCE – 1800 CE** (Princeton: Princeton University Press, 2009)

## **Algebra and Number Theory**

B. L. van der Waerden, **A History of Algebra from al-Khwarizmi to Emmy Noether** (New York: Springer, 1985)

Lubos Novy, **Origins of Modern Algebra** (Prague: Academia Publishing House, 1973)

Hans Wussing, **The Genesis of the Abstract Group Concept** (Cambridge: MIT Press, 1984)

André Weil, **Number Theory: An Approach through History from Hammurapi to Legendre** (Boston: Birkhäuser, 1984)

Leo Corry, **Modern Algebra and the Rise of Mathematical Structures** (Boston: Birkhäuser, 1996)

Helena M. Pycior, **Symbols, Impossible Numbers, and Geometric Entanglements: British Algebra Through the Commentaries on Newton's *Universal Arithmetick*** (Cambridge: Cambridge University Press, 1997)

Jacqueline A. Stedall, **A Discourse Concerning Algebra: English Algebra to 1685** (Oxford: Oxford University Press, 2002)

Israel Kleiner, **A History of Abstract Algebra** (Boston: Birkhäuser, 2007)

## **Geometry**

Carl Boyer, **History of Analytic Geometry** (New York: Scripta Mathematica, 1956)

Jeremy Gray, **Ideas of Space: Euclidean, Non-Euclidean and Relativistic** (Oxford: Clarendon Press, 1989)

Boris Rosenfeld, **A History of Non-Euclidean Geometry: Evolution of the Concept of a Geometric Space**, trans. by Abe Shenitzer (New York: Springer, 1988)

Michael J. Crowe, **A History of Vector Analysis** (New York: Dover, 1985)

Henk J. M. Bos, **Redefining Geometrical Exactness: Descartes' Transformation of the Early Modern Concept of Construction** (New York: Springer, 2001)

Marvin Jay Greenberg, **Euclidean and Non-Euclidean Geometries: Development and History** (New York: Freeman, 2008)

Jeremy Gray, **Worlds Out of Nothing: A Course in the History of Geometry in the 19th Century**, (New York: Springer Verlag, 2007)

Glen Van Brummelen, **The Mathematics of the Heavens and the Earth: The Early History of Trigonometry**, (Princeton: Princeton University Press, 2009)

## **Calculus and Analysis**

Carl Boyer, **The History of the Calculus and its Conceptual Development** (New York: Dover, 1959)

Margaret E. Baron, **The Origins of the Infinitesimal Calculus** (Oxford: Pergamon Press, 1969)

C. H. Edwards, **The Historical Development of the Calculus** (New York: Springer, 1979)

Ivor Grattan-Guinness, ed. **From the Calculus to Set Theory, 1630- 1910: An Introductory History** (London: Duckworth, 1980)

Umberto Bottazzini, **The Higher Calculus: A History of Real and Complex analysis from Euler to Weierstrass** (New York: Springer, 1986)

Ivor Grattan-Guinness, **The Development of the Foundations of Mathematical Analysis from Euler to Riemann** (Cambridge: MIT Press, 1970)

Judith V. Grabiner, **The Origins of Cauchy's Rigorous Calculus** (Cambridge: MIT Press, 1981)

Niccolò Guicciardini, **Reading the *Principia*: The Debate on Newton's Mathematical Methods for Natural Philosophy from 1687 to 1736** (Cambridge: Cambridge University Press, 1999)

## **Probability and Statistics**

F. N. David, **Games, Gods, and Gambling** (New York: Hafner, 1962)

Ian Hacking, **The Emergence of Probability** (Cambridge: Cambridge University Press, 1975)

Ian Hacking, **The Taming of Chance** (Cambridge: Cambridge University Press, 1990)

Stephen M. Stigler, **The History of Statistics** (Cambridge: Harvard University Press, 1986)

Lorraine Daston, **Classical Probability in the Enlightenment** (Princeton: Princeton University Press, 1988)

Theodore M. Porter, **The Rise of Statistical Thinking** (Princeton: Princeton University Press, 1986)

Gerd Gigerenzer et al, **The Empire of Chance: How Probability Changed Science and Everyday Life** (Cambridge: Cambridge University Press, 1989)

Lorenz Krüger, Lorraine J. Daston, and Michael Heidelberger, eds., **The Probabilistic Revolution** (2 volumes) (Cambridge: MIT Press, 1987)

Anders Hald, **A History of Probability and Statistics and Their Applications before 1750** (New York: Wiley, 1990)

Anders Hald, **A History of Mathematical Statistics from 1750 to 1930** (New York: Wiley, 1998)

James Franklin, **The Science of Conjecture: Evidence and Probability before Pascal** (Baltimore, Johns Hopkins University Press, 2001)

## **Foundations of Mathematics**

Gregory H. Moore, **Zermelo's Axiom of Choice: Its Origins, Development, and Influence** (New York: Springer, 1982)

Alejandro R. Garciadiego, **Bertrand Russell and the Origins of the Set-Theoretic "Paradoxes"** (Boston: Birkhäuser, 1992)

I. Grattan-Guinness, **The Search for Mathematical Roots, 1870-1940: Logics, Set Theories and the Foundations of Mathematics from Cantor Through Russell to Gödel** (Princeton: Princeton University Press, 2000)

José Ferreirós, **Labyrinth of Thought: A History of Set Theory and Its Role in Modern Mathematics** (Boston: Birkhäuser, 2007)

## **Ethnomathematics**

Claudia Zaslavsky, **Africa Counts: Number and Pattern in African Culture**, 3<sup>rd</sup> edition (Chicago: Lawrence Hill Books, 1999)

Marcia Ascher, **Ethnomathematics: A Multicultural View of Mathematical Ideas** (Pacific Grove, Ca.: Brooks/Cole, 1991)

Marcia Ascher, **Mathematics Elsewhere: An Exploration of Ideas Across Cultures** (Princeton: Princeton University Press, 2002)

Helaine Selin, ed., **Mathematics Across Cultures: The History of Non-Western Mathematics** (Dordrecht: Kluwer, 2000)

## **Euler**

Robert Bradley and C. Edward Sandifer, eds., **Leonhard Euler: Life, Work and Legacy** (Amsterdam: Elsevier, 2007)

William Dunham, ed., **The Genius of Euler: Reflections on his Life and Work** (Washington: MAA, 2007)

C. Edward Sandifer, **The Early Math of Leonhard Euler** (Washington: MAA, 2007)

C. Edward Sandifer, **How Euler Did It** (Washington: MAA, 2007)

N. N. Bogolyubov, G. K. Mikhailov, & A. P. Yushkevich, eds., **Euler and Modern Science**, Robert Burns, trans. (Washington: MAA, 2007)

Robert E. Bradley, Lawrence A. D'Antonio, C. Edward Sandifer, eds., **Euler at 300: An Appreciation** (Washington: MAA, 2007)

## Recent “Popular” Books in the History of Mathematics

Aczel, Amir. *Descartes’ Secret Notebook: A True Tale of Mathematics, Mysticism, and the Quest to Understand the Universe*. New York: Broadway Books, 2005.

Alder, Ken. *The Measure of All Things: The Seven-Year Odyssey and Hidden Error That Transformed the World*. New York: Free Press, 2003.

Bernstein, Peter. *Against the Gods: The Remarkable Story of Risk*. New York: John Wiley, 1996.

Derbyshire, John. *Prime Obsession: Bernhard Riemann and the Greatest Unsolved Problem in Mathematics*. Washington: Joseph Henry Press, 2003.

\_\_\_\_\_. *Unknown Quantity: A Real and Imaginary History of Algebra*. Washington: Joseph Henry Press, 2006.

Devlin, Keith. *The Unfinished Game: Pascal, Fermat and the Seventeenth Century Letter that Made the World Modern*. Xxx: Basic Books, 2008

Dunham, William. *The Calculus Gallery: Masterpieces from Newton to Lebesgue*. Princeton: Princeton University Press, 2005.

Field, J. V. *Piero della Francesca: A Mathematician’s Art*. New Haven: Yale University Press, 2005.

Gilder, Joshua and Anne-Lee. *Heavenly Intrigue: Johannes Kepler, Tycho Brahe, and the Murder Behind One of History’s Greatest Scientific Discoveries*. New York: Doubleday, 2004.

Gray, Jeremy. *János Bolyai, Euclid, and the Nature of Space*. Cambridge: MIT Press, 2003.

Havil, Julian. *Gamma: Exploring Euler’s Constant*. Princeton: Princeton University Press, 2003.

Hidetoshi, Fukagawa and Tony Rothman. *Sacred Mathematics: Japanese Temple Geometry*. Princeton: Princeton University Press, 2008.

Kaplan, Robert. *The Nothing That Is: A Natural History of Zero*. Oxford: Oxford University Press, 2000.

Livio, Mario. *The Golden Ratio: The Story of Phi, the World’s Most Astonishing Number*. New York: Broadway Books, 2002.

- \_\_\_\_\_ *The Equation That Couldn't Be Solved (How Mathematical Genius Discovered the Language of Symmetry)*. New York: Simon and Schuster, 2005.
- Maor, Eli. *To Infinity and Beyond: A Cultural History of the Infinite*. Princeton: Princeton University Press, 1991.
- \_\_\_\_\_ *e: The Story of a Number*. Princeton: Princeton University Press, 1994.
- \_\_\_\_\_ *Trigonometric Delights*. Princeton: Princeton University Press, 1998.
- \_\_\_\_\_ *The Pythagorean Theorem: A 4000-year History*. Princeton: Princeton University Press, 2007.
- Mazzotti, Massimo. *The World of Maria Gaetana Agnesi, Mathematician of God*. Baltimore: Johns Hopkins University Press, 2007.
- Nahin, Paul J. *An Imaginary Tale: The Story of  $\sqrt{-1}$* . Princeton: Princeton University Press, 1998.
- \_\_\_\_\_ *When Least Is Best: How Mathematicians Discovered Many Clever Ways to Make Things as Small (or as Large) as Possible*. Princeton: Princeton University Press, 2003.
- \_\_\_\_\_ *Dr. Euler's Fabulous Formula: Cures Many Mathematical Ills*. Princeton: Princeton University Press, 2006.
- Netz, Reviel and William Noel. *The Archimedes Codex*. London: Weidenfeld and Nicolson, 2007.
- Pasles, Paul. *Benjamin Franklin's Numbers: An Unsung Mathematical Odyssey*. Princeton: Princeton University Press, 2007.
- Pesic, Peter. *Abel's Proof: An Essay on the Sources and Meaning of Mathematical Unsolvability*. Cambridge: MIT Press, 2003.
- Rockmore, Dan. *Stalking the Riemann Hypothesis: The Quest to Find the Hidden Law of Prime Numbers*. New York: Pantheon Books, 2005.
- Salsburg, David. *The Lady Tasting Tea: How Statistics Revolutionized Science in the Twentieth Century*. New York: W. H. Freeman, 2001.
- Seife, Charles. *Zero: The Biography of a Dangerous Idea*. New York: Viking Press, 2000.
- Singh, Simon. *Fermat's Enigma: The Epic Quest to Solve the World's Greatest Mathematical Problem*. New York: Walker and Company, 1997.



\_\_\_\_\_ *The Code Book: The Science of Secrecy from Ancient Egypt to Quantum Cryptography*. New York: Anchor Books, 1999.

Sobel, Dava. *Longitude: The True Story of a Lone Genius Who Solved the Greatest Scientific Problem of His Time*. New York: Walker and Company, 1995.

Wakefield, Julie. *Halley's Quest: A Selfless Genius and His Troubled Paramore*. Washington: Joseph Henry Press, 2005.

Wilson, Robin. *Four Colours Suffice: How the Map Problem Was Solved*. London: Penguin Books, 2002.

More books can be found in the Book List in the *Notices of the American Mathematical Society* and in MAA Reviews, reachable through MAA Online.

## Student Projects

It is virtually standard in a history of mathematics course to assign a written project, or term paper. Since most mathematics professors have little experience in dealing with term papers, it is probably useful to consult with members of your English department when you do this for the first time. In giving an assignment for a term paper, you should be as specific as possible. In particular, you should establish a timetable for the choice of a topic, for an outline, for a list of references, and, if possible, for a first draft. Then you need to respond to each submission in a timely manner to insure that you students have time to improve their work.

Perhaps the most difficult part of assigning a term paper is helping the students with a choice of topic. Many students will pick an overly broad topic and will then find the information available to be overwhelming. You should encourage students to narrow their focus to a topic of manageable size, one for which your library has sufficient material. In fact, if your library is particularly strong in a certain area --- or has a good collection of rare books --- you should strongly suggest that the students choose topics that will make use of this resource. This is also true if your students have easy access to other academic libraries with specialized collections. Naturally, students can use the Internet for research, but this is sometime risky, as information on the Internet may not be reliable.

It is a good idea to pass out a list of possible projects to the students to give them guidance. You may want to require that, since this is a course in the History of Mathematics, the paper must involve history and it must involve mathematics. For example, if a student writes about an individual, it is not sufficient just to mention a whole bunch of mathematical topics the individual worked on. The student must actually explain enough of the mathematics so that one of their peers could understand it (and so that you have evidence that they have understood the mathematics they are writing about).

Obviously, there are a wide variety of topics that students could pick, but they may be divided roughly into four categories. First, they may write about the history of a particular mathematical topic. Second, they could write about an individual and his/her contributions. Third, if the resources are available, a student could delve into an original source and discuss its contents and influence. Finally – and this particularly applies to students who are prospective teachers – they could deal with the use of the history of mathematics in teaching.

Here are some suggestions for student projects dealing with a particular topic:

- The conic sections
- Durer's Polyhedra
- The Four Color Problem
- Holbein's Ambassadors
- Mathematics in Encyclopedias
- Geometric Algebra
- The regular polygons
- Changes in geometry textbooks over the ages

- Perspective in art and projective geometry
- Paradoxes in mathematics from Zeno to Russell
- The "fundamental" theorems of mathematics
- The change in the conception of "number" from the Greek period to the modern one
- The notion of function through the centuries
- Discussion of a mathematical controversy; e.g. Zeno's school vs. the Aristotelians on indivisibles; Newton vs. Berkeley on infinitesimal
- Mathematics in sub-Saharan Africa
- Mathematics in pre-Columbian North America
- The ethnomathematics of a particular culture
- The beginnings of chaos theory

If students want to write about an individual, it would probably be best to take someone other than Euler, Gauss, Newton, etc. So some suggestions include:

- Benjamin Banneker and his almanacs
- Charles Babbage, computer pioneer
- Julia Robinson, first woman president of the AMS
- Ada Lovelace, early programmer
- Bourbaki
- Alan Turing

Here are some suggestions for student projects designed for prospective teachers:

- Compare the Babylonian, Mayan, and Hindu-Arabic place-value systems in their historical development and their ease of use. Devise a series of lessons to teach place value using these systems.
- Devise a series of geometrical models designed to derive area and volume formulas for geometrical figures, including circles, spheres, pyramids, cones, and truncated pyramids.
- Look at the history of formulas for sums of integral powers. Consider the use of these in developing volume formulas and devise several lessons to use this material in an introductory calculus class.
- Consider the history of the limit concept from Eudoxus to the mid-eighteenth century. In particular, consider Berkeley's criticisms and Maclaurin's response. Devise some lessons explaining the limit concept using this history.
- Consider the history of the idea of a matrix and apply this in the teaching of linear algebra.
- Discuss the development of spherical trigonometry from the Greeks to the Islamic mathematicians. Discuss how and whether one can use this material in trigonometry classes.
- Discuss how the history of the solution of cubic equations from the Islamic period through the work of Lagrange can be used in algebra classes at various levels.
- Discuss how the history of the notion of infinity can be used in mathematics classes at various levels.

- Compare the teaching of algebra (or geometry) in the eighteenth century and the twentieth by studying textbooks.

Fernando Q. Gouvêa has a nice bunch of suggestions that use original sources. The sources mentioned are available in his library at Colby College and some of the suggestions are directly related to the college. To make your own list, you will have to become familiar with your local resources.

- **Madame de Châtelet:** find out more about Émilie de Châtelet, her mathematics, and her influence. How influential was she? Did she ever do any original work in physics or mathematics? A possible starting point is the article “Émilie du Châtelet and the Gendering of Science”, by Mary Terrall, *History of Science*, 33 (1995), 283-310.
- **Smallpox inoculation** was very controversial in eighteenth-century France, and at least part of the controversy was put in mathematical terms: what is the real advantage of inoculation? Can one determine the probability that it will improve one’s life expectancy? Do I benefit personally from an increase in the life expectancy of the overall population? Both Daniel Bernoulli and D’Alembert wrote about this problem. I have a copy of (a translation of) their papers. Reading those isn't really easy, but would be the right place to start. There's more on the topic in Hankins and in Daston's *Classical Probability in the Enlightenment*.
- **Maupertuis and the shape of the Earth:** we spent a class session discussing this one, but there's clearly a lot more to say. Exactly what did the expeditions measure? If the Earth is flattened, do we expect a degree of longitude near the pole to be longer or shorter than at the equator? And what about the seconds pendulum: what does *that* have to do with it? Possible starting points: there are two papers one could start from. One is by J. L. Greenberg and appeared in *Archive for the History of the Exact Sciences* (not in the library, but I have a copy of the paper). The other is by R. Iliffe, and appeared in *History of Science* in 1993 (this one is in the library). We also have Greenberg's book on the problem.
- **A great textbook and its influence:** Euler's *Elements of Algebra* was an immensely influential textbook. What is in the book? Does it read like a modern algebra text? Can one still find traces of its influence in modern textbooks? The library has a copy of Euler's book (translated into English), and one would start by looking at its table of contents and reading a well-chosen section. Then one could go on to compare that section with a related section in more recent books.
- **The Ladies Diary:** the short article we read had more questions in it than answers. Find out more about *The Ladies Diary*. Who read it? What was in it? Why did the male readers “take it over” at the end of the eighteenth century? A possible starting point is the article “Female Philomaths”, by Ruth and Peter Wallis, *Historia Mathematica*, 7(1980), pages 57-64. The Colby library has this journal (in Miller Library).
- **Montucla's History of Mathematics:** one of the treasures in Colby's Special Collections is a copy of the first modern attempt at writing a history of mathematics, by Jean E. Montucla. The copy we have is the second edition, with additions by J. de Lalande, which was published in 1799. Two possible projects suggest themselves: one could write

about the book itself (what is in it, was it influential, how does it fit in the history of histories of science), or one could choose a topic and compare what Montucla and Lalande say with what a modern history says. In the latter case, the best would be to stick with one of “our” topics (e.g., the notion of a differential, probability, number theory, Newtonian mechanics).

- **Hutton's Dictionary:** another treasure we own is a copy of the *Philosophical and Mathematical Dictionary*, by Charles Hutton. The edition we have is from 1815. It might be interesting to take one of “our” topics and see how Hutton perceived it at the beginning of the nineteenth century.
- **Monge and Descriptive Geometry:** there are lots of topics we won't get to talk about in class, and Gaspard Monge and his “Descriptive Geometry” is one of them. But there's lots to say about both the man and the mathematics. Monge was one of the most political of the late eighteenth century mathematicians, so this would have lots of connections to the French Revolution material from H177. A possible starting point would be an old essay by D. E. Smith on “Gaspard Monge, politician”, that you can find in a book called *The Poetry of Mathematics and other essays* (in the Colby library). Of course, since this essay is over half a century old, you'll want to look at other sources too.
- **Saccheri and the parallel postulate:** another topic we won't really get to is the work on Euclid's “fifth postulate”. This became a hot topic in the nineteenth century, but Girolamo Saccheri's book *Euclides Vindictatus* (the title is sometimes given as *Euclid Freed From Every Flaw*), which first appeared in 1733, is an important milestone in the history of geometry. Explaining the goals and the achievements of Saccheri would make a great term paper. Our library has an English edition of Saccheri, and the controversy over the parallel postulate is discussed in pretty much every book on the history of mathematics, so there's lots of material to work with. There are also other mathematicians who worked on this problem, notably Lambert and Legendre; you can find extracts from their work in the Fauvel-Gray sourcebook.

## **SUGGESTED TOPICS FOR THEMED VERSIONS OF A HISTORY OF MATHEMATICS COURSE**

### **I Algebra emphasis**

1. We will see how ancient peoples solved various types of problems through the use of what we call linear and quadratic equations. We will study the origins of the quadratic formula in geometry and consider why solving quadratic equations was important in ancient Babylonia.
2. The Euclidean version of algebra is generally called “geometric algebra.” We will look at the development of this algebra in Greece and its relationship to its Babylonian predecessors. We will then consider the entirely different algebra of Diophantus.
3. After a brief look at algebraic ideas in India and China, we consider the first true algebra text, written by al-Khwarizmi in the ninth century. We will consider this and related texts in detail to see how the Islamic mathematicians understood algebra and how they developed the laws of exponents and the basic principles of polynomial algebra.
4. We will consider the development of the idea of mathematical induction in the Islamic and Jewish cultures of the middle ages, especially in connection with combinatorial ideas.
5. Cubic equations were solved geometrically by Omar Khayyam in the eleventh century, through the use of nascent ideas of calculus by Sharaf al-Din al-Tusi in the twelfth century, and then algebraically by various Italian mathematicians in the sixteenth century. We will look at their methods and then see how the concept of a complex number grew out of the algebraic solution techniques. We also consider some of the reasons for the interest in solving equations of second and third degree.
6. Solution techniques for cubic and quartic equations were investigated in detail in the eighteenth century at the same time as attempts were being made to solve equations of higher degree. These investigations were one of the sources of the idea of a group. We will consider other sources of this notion as well.
7. We look at the late nineteenth and early twentieth century development of various other mathematical structures, including rings, fields, and vector spaces. We consider the reasons for the growing abstractness of algebra.
8. We conclude with a rapid survey of the developments leading to Wiles’ proof of Fermat’s Last Theorem.

## II Calculus emphasis

1. We consider how ancient peoples calculated areas and volumes of various figures and try to understand how and why they arrived at their procedures.
2. We will study Greek geometry, paying particular attention to the development of the proof process. We consider the idea of calculating areas and volumes by approximating them through the areas and volumes of simpler figures and learn how this was accomplished and how the results were proved by Greek mathematicians and later by Islamic mathematicians.
3. The idea of a function grew up with the use of trigonometric ideas to measure the heavens. We will therefore study ancient astronomy and follow the growth and travels of trigonometry (both plane and spherical) from Greece and Egypt to India, then to China, to the Middle East, and finally back to Europe. We then consider the work of Copernicus and Kepler on the development of heliocentric astronomy, and the work of Galileo on kinematics.
4. Although curves were drawn in ancient times, a new era in their study began with the invention of analytic geometry in the seventeenth century in France. We consider the work of both Fermat and Descartes and the development of new techniques by them and others to deal with maxima and minima, techniques which eventually grew into our algorithms for derivatives.
5. We next consider the development of algorithms for determining areas and lengths. These are related to ideas about power series, some of which have precursors in medieval India. We consider how calculations of arc length led to the fundamental theorem of calculus.
6. Why are Newton and Leibniz considered the inventors of calculus? We look at the accomplishments of both men, including Newton's work on celestial mechanics.
7. We deal with developments in calculus in the eighteenth century, including the criticism of the foundations of calculus by Berkeley, the responses by Maclaurin and d'Alembert, and the continued development of calculus techniques by Johann Bernoulli, Leonhard Euler, and others.
8. The rigorous development of calculus took place in the nineteenth century. We look at Lagrange's attempt to found calculus on the notion of a power series, at Cauchy's notions of limits and continuity, and at the idea of a Fourier series and the reconsideration of the notion of a function. We then deal with the arithmetization of analysis in the work of Dedekind, Cantor, and Weierstrass.

9. We conclude with a brief history of the theory of differential equations, concluding with Poincare's work on the three-body problem, the origins of chaos theory.

### III Geometry emphasis

1. We will see how ancient peoples calculated areas and volumes of various figures and try to understand how and why they arrived at their procedures.
2. We will study Greek geometry, paying particular attention to the development of the ideas in Euclid's *Elements*, including the notion of an axiomatic system and logical proof. We will also consider the connections between Greek geometry and the geometry of the ancient civilizations of the Middle East. We will then look at the development of the notion of a limit process, originating in the work of Eudoxus and continuing in the work of Archimedes and later Islamic mathematicians.
3. We will consider the development of the theory of conic sections and study the properties of these curves.
4. Astronomy was important to ancient peoples, so we will study that field along with trigonometry, the major mathematical tool needed to explore it. We will follow the growth and travels of trigonometry from Greece and Egypt to India, to China, to the Middle East, and back to Europe.
5. We will consider the mathematics of surveying in China and India and look at the relationship between ancient methods and trigonometrical methods throughout the centuries.
6. Analytic geometry was developed by two French mathematicians in the early eighteenth century, Descartes and Fermat. We will consider the similarities and differences of their approaches to the subject and consider how their methods were used by Newton and Leibniz in developing the calculus.
7. We will look at the attempts to prove Euclid's parallel postulate over the centuries, attempts which finally led to the development of non-Euclidean geometry in the nineteenth century.
8. We will conclude with a study of the generalization of geometric methods from two and three dimensions to  $n$  dimensions, generalizations connected with the idea of a vector space and, more generally, with the concept of a mathematical structure.



## Scheduling a course using Katz, *A History of Mathematics, Brief Edition*

This brief edition is designed so that most of the text can in fact be covered in the standard one semester course generally given in the history of mathematics. In particular, it is important in any such course to reach the final chapter on Aspects of the Twentieth Century. It is certainly important for mathematics students, especially those who are intending to be secondary teachers, to realize that the mathematical enterprise is an ongoing one, that old problems continue to be attacked, and that some very difficult ones do eventually yield to creative mathematicians. Nevertheless, the book is designed so there are alternative pathways through it. For example, it is certainly possible to go through the book in order, chapter by chapter. On the other hand, it is possible to trace, in order, various themes. That is, one might first cover the history of geometry, then the history of algebra, then the history of calculus, and even the history of probability and statistics. The chapter and section headings will help you do this.

Here are two possible schedules for a course with 40 class meetings.

### SCHEDULE OF A COURSE PRESENTED CHRONOLOGICALLY

1. Ancient Egypt - Rhind and Moscow papyri (1.1)
2. Mesopotamian mathematics - numbers and geometry (1.2.1-1.2.3)
3. Mesopotamian mathematics - algebra (1.2.4-1.2.5)
4. Beginnings of Greek mathematics (2.1)
5. Euclid's *Elements* - Book I and the Pythagorean Theorem (2.2.1)
6. Euclid's *Elements* - Geometric algebra and the pentagon construction (2.2.2-2.2.3)
7. Euclid's *Elements* - Ratio, proportion, and incommensurability (2.2.4, 2.2.6)
8. Archimedes (3.1)
9. Apollonius (3.2)
10. Introduction to Greek astronomy (3.3.1-3.3.2)
11. Ptolemy - astronomy and trigonometry (3.3.3-3.3.5)
12. Diophantus (4.1)
13. Chinese mathematics - numbers and geometry (5.1-5.2)
14. Chinese mathematics - algebra (5.3-5.4)
15. Indian mathematics - geometry and algebra (6.1-6.3)
16. Indian mathematics - trigonometry (6.5)
17. Islamic mathematics - algebra (7.2)
18. Islamic mathematics - geometry and trigonometry (7.4-7.5)
19. Medieval Europe - combinatorics and algebra (8.2-8.3)
20. Renaissance Europe - the solution of the cubic equation (9.1)
21. Renaissance Europe - Trigonometry and logarithms (9.2-9.3)
22. Renaissance Europe - astronomy and physics (9.4)
23. Seventeenth century - algebra and analytic geometry (10.1-10.2)

24. Seventeenth century - tangents and areas (11.1-11.2)
25. Seventeenth century - the calculus according to Newton (11.4.1-11.4.2)
26. Seventeenth century - Newton's physics (11.4.3)
27. Seventeenth century - the calculus according to Leibniz (11.5)
28. Eighteenth century - differential equations (12.1)
29. Eighteenth century - foundations of the calculus (12.4)
30. Eighteenth century - algebra and number theory (14)
31. Eighteenth century - the parallel postulate (15.1)
32. Nineteenth century - the beginning of structure in algebra (16.2-16.3)
33. Nineteenth century - matrices and linear equations (16.4)
34. Nineteenth century - rigor in analysis (17.1)
35. Nineteenth century - the arithmetization of analysis (17.2)
36. Nineteenth century - complex analysis (17.3)
37. Nineteenth century - the development of non-Euclidean geometry (19.1)
38. Twentieth century - growth of abstraction (20.1)
39. Twentieth century - major new results (20.2-20.3)
40. Twentieth century - computers and their influence (20.4)

#### SCHEDULE OF A COURSE PRESENTED THEMATICALLY

1. Ancient geometry — areas and volumes (1.1.4, 1.2.3)
2. Ancient algebra - Mesopotamian solutions to quadratic equations (1.2.5)
3. Beginnings of Greek mathematics (2.1)
4. Euclid's *Elements* - Book I and the Pythagorean Theorem (2.2.1)
5. Euclid's *Elements* - Geometric algebra and the pentagon construction (2.2.2-2.2.3)
6. Euclid's *Elements* - Ratio, proportion, and similarity (2.2.4)
7. Apollonius and the conic sections (3.2)
8. Ptolemy and trigonometry (3.3.3-3.3.4)
9. Spherical trigonometry and the solution of astronomical problems (3.3.5, 7.5.2)
10. Indian geometry and trigonometry (6.2, 6.5)
11. Geometry in the Middle Ages and Renaissance (8.1, 9.2)
12. Trigonometry in the Renaissance (9.3)
13. The parallel postulate in the eighteenth century (15.1)
14. The development of non-Euclidean geometry (19.1)
15. Diophantus and Greek algebra (4.1)
16. Solving equations in China (5.3)
17. Algebra in India (6.3)
18. Islamic algebra (7.2)

19. Medieval algebra (8.3)
20. Algebra in the Renaissance - the solution of the cubic equation (9.1)
21. Algebraic symbolism and the theory of equations (10.1)
22. Analytic geometry - Fermat and Descartes (10.2)
23. Eighteenth century equation solving (14.1, 14.2)
24. Nineteenth century equation solving - beginnings of Galois theory (16.2)
25. Groups and fields - algebraic structure (16.3)
26. Twentieth century algebra - Fermat's Last Theorem and Finite Simple Groups (20.2.1, 20.2.2)
27. Euclid and the method of exhaustion (2.2.6)
28. Archimedes and determination of areas and volumes (3.1)
29. Areas and volumes in Islam (7.4.2)
30. Tangents and extrema in the seventeenth century (11.1)
31. Areas and volumes — Fermat, Roberval, Pascal (11.2)
32. Rectification of curves and the fundamental theorem (11.3)
33. The calculus according to Newton (11.4)
34. The calculus according to Leibniz (11.5)
35. Differential equations in the eighteenth century (12.1)
36. The foundations of the calculus (12.4)
37. Limits, continuity, and convergence (17.1.1-17.1.3)
38. The rigorization of the calculus (17.1.4-17.1.8)
39. The arithmetization of analysis (17.2)
40. Complex analysis and the Riemann Zeta function (17.3)

## Self Evaluation of Your Course

To improve your history of mathematics course, you will have to develop the habit of evaluating your performance yourself. If you develop the habit of thinking carefully about what you have done and how you can improve and continually reflect on these issues, then it is likely that your course will continually improve.

### After Each Class

After each class you should make notes about how the class went, what went well, and what you can do to improve. It will take discipline to do this, for it is much easier to just stuff everything from the day in a folder and forget about it until the next time you teach the course. But it is well worth your time to organize the material in the folder so that it is ready to go again and then spend some time making notes to yourself for the next time. There is no question that it takes considerable diligence to do what is being suggested. But it is very helpful.

Here are some things you should do after each class:

- Record what material you covered in the class. Make notes on your classnotes about things that you did and didn't cover.
- Make some honest comments in your class notes about what went well and about what should have been better prepared. Devise a plan for improving weak points.
- Be sure that your class notes list all of the references you consulted in preparing your lecture, including the page numbers consulted.
- Were there journal articles that you wanted to consult while preparing class and that were not available in your library? If so, order them via interlibrary loan now so that you will have them later.
- List the slides that you used in class. What comments did they evoke from the students? What slides did you wish you had? Made a list of the kinds of slides that you are looking for and then keep them in mind as you look at various sources during the term. Perhaps you will find what you want. If not, post a note to a history email list asking where such materials can be found.
- What questions did the students ask in class? Which of these questions were you unable to answer satisfactorily? Can you do the research necessary to answer them by the next class? If so, do it. If the questions were factual, an email list provides a good way to answer them. This saves time during the next class, provides an answer while the student who asked the question still has it in mind, and convinces the whole class that you care about their interests.
- What questions occurred to you during the class? Were there historical questions that you wished you knew the answers to? Were there mathematical issues where you wished you had more detail?
- Did any exam questions occur to you while lecturing? Was there a point that you made in class that you put special stress on? If so, perhaps it should be developed into a potential exam question.
- Did any ideas for student papers occur to you?

**SYLLABUS**  
**MATH 274 – HONR 228I**  
**HISTORY OF MATHEMATICS**  
**SPRING, 2003**

**Course Description:** This course will provide an overview of aspects of the history of mathematics from its beginnings in concrete problem solving in ancient times through the development of abstraction in the nineteenth and early twentieth centuries. The course will consider both the growth of mathematical ideas and the context in which these ideas developed, in various civilizations around the world. Attention will be paid to how the history of mathematical ideas is important in the teaching of these ideas in both secondary school and college.

**Textbook:** Katz, Victor J. *A History of Mathematics: An Introduction* (2<sup>nd</sup> ed.) (Reading, Mass.: Addison-Wesley, 1998).

**Additional Readings:** Additional material, primarily original sources in mathematics, will be distributed regularly.

**Course Outline:** There will be 10 units in the course, lasting from 2 to 5 class periods each. The first and last date of each unit is indicated below, along with the text assignment and the additional readings for that unit.

1. **Jan 28 – Feb 4. Ancient Mathematics:** We will see how and why ancient peoples solved various types of mathematical problems, putting these problems into the social context of the particular civilizations, especially Egypt, Mesopotamia, and China. We will consider who the people were who “did” mathematics and what their role was in the society. We will look at how ancient peoples calculated areas and volumes of various figures and try to understand how and why they arrived at their procedures. We will also study the origins of the quadratic formula in geometry and consider why solving quadratic equations was important in ancient Mesopotamia.

Text: Chapter 1. Additional readings: Two Babylonian tablets, two problems from the Moscow Papyrus, two problems from the *Nine Chapters on the Mathematical Art*.

2. **Feb 6 – Feb 20. Greek Geometry:** We will study Greek geometry, paying particular attention to the development of the proof process and how this relates to the position of mathematicians in Greek society.. We consider the idea of calculating areas and volumes by approximating them through the areas and

volumes of simpler figures and learn how this was accomplished and how the results were proved by Greek mathematicians. We will also look at Greek “geometric algebra” and consider its relationship to Mesopotamian algebra. We conclude this section with a brief look at the development of conic sections.

Text: Chapters 2, 3. Additional readings: Selected propositions from Euclid’s *Elements* and Archimedes’ *Method*.

3. **Feb 25 – Mar 4. Trigonometry and its Travels:** The idea of a function grew up with the use of trigonometric ideas to measure the heavens. We will therefore study ancient astronomy and follow the growth and travels of trigonometry (both plane and spherical) from Greece and Egypt to India, then to China, to the Middle East, and finally back to Europe. We consider the importance of both astronomy and trigonometry in each of these civilizations.

Text: Sections 4.1, 4.2, 6.1, 6.2, 6.5, 6.6, 7.5, 8.1, 10.3. Additional readings: Selections from Ptolemy’s *Almagest*.

4. **Mar 6 – Mar 13. Islamic Algebra and Combinatorics:** We consider the first true algebra text, written by al-Khwarizmi in the ninth century. We will consider the reasons for the study of algebra in Islam and then see how the Islamic mathematicians understood the subject and how they developed the laws of exponents and the basic principles of polynomial algebra. We will also consider the development of the idea of mathematical induction in the Islamic and Jewish cultures of the middle ages, especially in connection with combinatorial ideas.

Text: Sections 7.2, 7.3, 8.2, 8.3. Additional readings: Selections from al-Khwarizmi’s *Algebra* and Levi ben Gerson’s *Maasei Hoshev*.

5. **Mar 18 – Apr 1. Cubic Equations:** Cubic equations were solved geometrically by Omar Khayyam in the eleventh century, through the use of nascent ideas of calculus by Sharaf al-Din al-Tusi in the twelfth century, and then algebraically by various Italian mathematicians in the sixteenth century. We will look at their methods and then see how the concept of a complex number grew out of the algebraic solution techniques. We also consider some of the reasons for the interest in solving equations of second and third degree in Europe in the Renaissance.

Text: Chapter 9; Section 11.2. Additional readings: Selections from Omar Khayyam’s *Algebra* and Gerolamo Cardano’s *Ars Magna*.

6. **Apr 3 – Apr 8. Analytic Geometry and the Beginnings of Calculus:** Although curves were drawn in ancient times, a new era in their study began with the invention of analytic geometry in the seventeenth century in France. We consider the work of both Fermat and Descartes and the development of new techniques by them and others to deal with maxima and minima, techniques which eventually

grew into our algorithms for derivatives. We will also look at the work of Fermat and others in working out algorithms for finding areas under certain curves.

Text: Sections 11.1, 12.1, 12.2 Additional readings: Selections from Fermat's *Introduction* and Descartes' *Geometry*.

7. **Apr 10 – Apr 17. The Invention of Calculus:** Why are Newton and Leibniz considered the inventors of calculus? We look at the accomplishments of both men in the context of their times. We will see that, although Newton did not develop the calculus in order to develop his celestial mechanics, he did make use of it in working out the details of the *Principia*. And we will also consider the reasons for Leibniz's interest in the subject and his use of it to solve numerous problems.

Text: Sections 12.3-12.6. Additional readings: Selections from Newton's *De Analysisi* and *Principia* and from Leibniz's first papers on calculus.

8. **Apr 22 – Apr 24. Calculus in the Eighteenth Century:** We consider developments in calculus in the eighteenth century, including the criticism of the foundations of calculus by Berkeley, the responses by Maclaurin and d'Alembert, and the continued development of calculus techniques by Johann Bernoulli, Leonhard Euler, and others. In particular, we will look at the problems that these mathematicians solved through the use of their new techniques.

Text: Chapter 13. Additional readings: Selections from Euler's *Introductio* and *Differential Calculus*.

9. **Apr 29 – May 6. Rigor in the Calculus:** The rigorous development of calculus took place in the nineteenth century. We look at Lagrange's attempt to found calculus on the notion of a power series, at Cauchy's notions of limits and continuity, and at the idea of a Fourier series and the reconsideration of the notion of a function. We will see how the changes in society wrought by the French Revolution impacted on the work of these men. We then deal with the arithmetization of analysis in the work of Dedekind, Cantor, and Weierstrass.

Text: Sections 14.4-14.5, 16.1-16.3. Additional readings: Cauchy's first proof using epsilon and delta.

10. **May 8 – May 13. The Beginnings of Abstract Algebra:** Solution techniques for cubic and quartic equations were investigated in detail in the eighteenth century at the same time as attempts were being made to solve equations of higher degree. These investigations were one of the sources of the idea of a group. We will consider other sources of this notion as well. We will see how this development led to the growth of abstraction in mathematics.

Text: Section 14.2, Chapter 15, Section 18.3.

**Course Organization:** There will be lectures and discussions of text material. At selected sessions, one student will be responsible for leading a discussion of one of the additional readings, based on questions distributed with the reading. For each unit of the course, each student will solve and hand in five problems of his or her choice from the relevant chapters or sections. You will get the most out of the course by solving the most difficult problems you can handle. A discussion question counts for two problems. Please indicate clearly the problems solved on your paper. Answers to discussion questions should be typed and should be no more than one page each. This assignment is due by the first day of the following course unit.

**Writing Assignments:** There are two major writing assignments. First, a book review of a book on the history of mathematics is due by March 13. A list of suggested books will be distributed, but you may choose another with the approval of the instructor. Guidelines for the review will also be distributed, but the length should be no more than 1500 words. Second, a research paper is due on May 13. Guidelines for the research paper will also be distributed, but I suggest that the paper be related to the book read for the book review. And since this is a course in the history of mathematics, the paper should include both history and mathematics. The exact balance will, of course, depend on the topic. Since many of you are prospective teachers of mathematics, the research paper may deal with the use of the history of mathematics in teaching a particular topic or set of topics at the elementary or secondary level. There is no fixed length for the research paper, because a topic generally has a “natural” length. However, the paper should not be shorter than five pages or longer than fifteen. You should give me a topic for the paper by March 4 (with at least three references indicated) and an outline by April 8. I will respond both to the topic and the outline with suggestions. You are encouraged to discuss your topic with me, as I can probably recommend references.

**Examinations:** There will be a midterm examination on March 20 and a final examination on Tuesday, May 20 from 10:30am to 12:30pm. Both exams will have a mixture of short answer and essay questions.

**Grading:** The various activities will be weighted as follows (1200 possible):

- Homework – 200
- Class Participation – 150
- Book Review – 150
- Research Paper – 300
- Midterm – 150
- Final Exam – 250

By percentage of total points earned, an A is 90-100%, a B is 80-89%, a C is 70-79%, a D is 60-69%, and an F is less than 60%.

**Student Expectations:** You are expected to come to class having done the required reading, participate actively in class discussion, and hand in assignments on time. You



are encouraged to work on homework problems, even discussion questions, with other students, but please write the results up individually.

**Religious Observances:** If your religion dictates that you cannot take an exam or hand in assigned work on a particular date, then contact me at the beginning of the semester to discuss alternatives. You are responsible for making these arrangements.

**Disabilities:** If you have some disability related to testing under the usual timed, in-class conditions, you may contact the office of Disabled Students Services (DSS) in Shoemaker. If they assess you as meriting private conditions and/or extra time, then you may arrange to take your tests at DSS, with extra time as they indicate. You must arrange this well in advance of a test.

**Honor Code:** The University has a nationally recognized Honor Code, administered by the Student Honor Council. The Code prohibits students from cheating on exams, plagiarizing papers, submitting the same paper for credit in two courses without authorization, buying papers, submitting fraudulent documents, and forging signatures. The Student Honor Council proposed and the University Senate approved an Honors Pledge. The University of Maryland Honor Pledge reads: "I pledge on my honor that I have not given or received any unauthorized assistance on this assignment/examination." Unless you are specifically advised to the contrary, the Pledge statement should be handwritten and signed on the front cover of all papers and exams submitted for evaluation in this course. Students who fail to write and sign the Pledge will be asked to confer with the instructor.

**Instructor:** Victor J. Katz, Visiting Professor of Mathematics

**Office:** MTH 2312

**Office Hours:** TTh 3:30-5:00 and by appointment

**Phone:** 301-405-5458

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# HISTORY OF MATHEMATICS

## Math 311. Spring 1998. Prof. Rickey

**Textbook:** *A History of Mathematics. An Introduction*, by Victor J. Katz, HarperCollins, 1993. Read it carefully before class so you are prepared to ask questions and to participate in discussions. Of course you should read it again later very carefully. We will cover most of the book, including some topics from the twentieth century.

**Office Hours:** I will be in my office, Room 406 MSC, Monday and Wednesday from 1:30 to 3:00. If my office hours change unexpectedly due to events beyond my control I will do my best to notify you by email. You can also talk to me after class. Other times are available by appointment. You are encouraged to stop in -- it is part of your education.

### Aims of the Course:

1. To give life to your knowledge of mathematics.
2. To provide an overview of mathematics -- so you can see how your various courses fit together and to see where they come from.
3. To teach you how to use the library and internet (important tools for life).
4. To show you that mathematics is part of our culture.
5. To indicate how you might use the history of mathematics in your future teaching.
6. To improve your reading and analytic skills, especially in a technical situation.
7. To improve your oral and written communication skills in a technical setting.

Remark: You should think about why you signed up for this course and what you hope to get out of it. If your goals are different than mine, let me know and perhaps I can accommodate you.

**Stress of the Course:** This course is designed as a survey of the history of mathematics. But far too much mathematics has been done in the past 4000 years to treat its entire history carefully, so we will concentrate on one theme: The development of calculus. To do this we will have to discuss the development of algebra, geometry, and trigonometry. Consequently we will discuss the history of most of the mathematics that is discussed in the high schools; this is intended to benefit the prospective teacher. We will not discuss many topics in modern mathematics.

**Automathography via email:** Each of you is required to get an email account so that I can communicate with you outside of class, and so that you can communicate with each other via a mail list called mathchat2. The first assignment involving email is to send me an automathography. You are to introduce your mathematical self to me. Tell me about the courses you have taken, what your favorites were, what you find hardest. Explain what your mathematical interests are, and what you plan to do after graduation. Reveal why you signed up for this course and what you expect to get out of it. If your aims for

taking this course are different than those stated above, please let me know. If you have any anxieties about this course, or any special problems or needs, let me know. You are encouraged to be creative in your response; don't be pedantic and just answer the questions asked above; include whatever you wish. There are three purposes for this assignment: to make sure that you can use email, to introduce yourself to me, and for me to get some idea about your writing skills. Send your response by email by Friday January 16 (Yes, I realize this is not a class day).

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Woops, forgot to include my email address. Here it is [well, my current email]: [fred-rickey@usma.edu](mailto:fred-rickey@usma.edu) .

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**First Library Assignment:** The purpose of these two assignments is to help you to learn your way around the library and the computer library catalogues. At first they may seem like busywork, but you will appreciate them when you are working on your paper.

For the first assignment you will be given the name of a mathematician and be asked to find out what you can about him or her. The questions asked on the sheet giving the name of your mathematician are designed to acquaint you with a few of the most useful reference works in the library dealing with the history of mathematics. Please answer the questions on this sheet and return it. [Each student was given a page with the name of [Your Mathematician](#) and questions to answer. They were also given a sample biography of [Al-Khwarizmi](#).]

Some of the questions ask you to look up works by and about the mathematician. Your records on these can be turned in on 4 by 6 index slips (cards are too bulky; other sizes are unacceptable), but I would strongly encourage you to send them to me via email. Each should contain the author's name, title, date of publication (the publisher and place of publication are usually not of much interest), the library call number and any other information that is of interest to you. You should use the following format (for a book):

Klein, Felix (1849--1925)

**The Evanston Colloquium. Lectures on Mathematics,**

New York: Macmillan, 1894

Call Number: BGSU: SL 510.8 K63e

English translation of twelve expository lectures given at Northwestern University in conjunction with the Chicago World's Fair in 1893. One contains a new proof of the transcendence of pi, a result needed to show that one cannot square the circle with straightedge and compass.

You should also write a short two page biography of the individual. Include information about early education and background, contributions to mathematics as well as other fields, and whatever other information you have found that is of interest (e.g., anecdotes).

You must supply references on your sources, as part of the intent of this exercise is for you to learn to deal with footnotes.

This biography should be posted on our class email list so that everyone in the class can read them. I encourage all students to read them carefully and make suggestions for improvement (this can be done off list, but do send me a copy). The purpose for posting all of these biographies is so that everyone can learn from them. After they have been posted for a week and people have had an opportunity to ask for clarification and to make suggestions, I will give you the opportunity to submit a revised copy for a grade. It is expected that many students will be asked to revise their paper.

You will be our class expert on this individual so be prepared to say something when the name comes up in class later. This assignment is due January 30. It is worth 50 points.

If there is sufficient interest we can convert these biographies into web pages. Yes, this would be for extra credit. Say up to 25 points, depending on the value added. Of course your name would go on the page.

Note: You are strongly encouraged to start a permanent file of library slips. This is a lifelong habit that is well worth developing. This is especially true of those of you who intend to become teachers, as you will have many occasions to want to have more information about a topic that you once read about. There are also several software packages available for keeping track of references (I use EndNote Plus), but I know of no freeware.

**Second Library Assignment:** The second library assignment is designed to acquaint you with the periodical literature. You are to turn in short synopses of ten papers on the **history of mathematics** (both words are crucial) that you have looked up and read. You should post this information to our class email list. The primary purpose of doing this is that it will provide everyone with lots of ideas for their major paper. Each is to contain complete bibliographic details: author, title (in quotes), periodical, volume, date (in parentheses following the volume number), and pages. The slip must also contain a short summary of the paper. The most interesting of these will be edited and posted on the web, so write your synopses with an eye towards encouraging your peers to read the paper. But be honest, if the paper is uninteresting, boring or not well written, say so.

A typical slip (for a journal article) is as follows:

Hogan, Edward  
1971 "Robert Adrain: American mathematician,"  
**Historia Mathematica**, vol. 4 (1971), pp. 157-172.

A very good, interesting biographical sketch concentrating on Adrain's publication of two journals, his teaching, and his mathematics. Very easy to read.

There are three ways to do this assignment and I encourage you to try them all:

1. Pick up one of the journals listed in the bibliography (to be distributed) and browse until you find a historical paper that interests you.
2. Pick a topic that interests you, look it up in Kenneth O. May's **Bibliography and Research Manual of the History of Mathematics** and then go find the paper.
3. Use one of the databases available through the library computer. I will bring a computer to class one day and discuss how you can do this.

The last two techniques will be the one you will have to use when writing your research paper, so I encourage you to do some of this. Another possibility is to follow up references that you encounter in your reading. Be sure to look in some old journals as they can be lots of fun.

The ten articles are to come from at least four different periodicals (books are not permitted), and deal with at least four different mathematical topics. Once an article has been posted it should not be duplicated; to prevent problems in this regard, you should post your work shortly after you have done it; don't wait till you get your synopses of all 10 articles completed.

This assignment is due February 6. It is worth 50 points.

You may find it easier to work on these two library assignments simultaneously. At the same time you should begin thinking about a topic for your research paper.

While doing these assignments you are strongly encouraged to browse in the library. Whenever you pick up a volume of a journal to look up one thing see what else it has of interest. Be sure to make slips on anything you find that interests you. Don't rely on your memory; I guarantee it will fail you.

**Internet Exercise:** The quality of the information on the internet is highly variable and judging its validity is a highly valuable skill. You will be given a sequence of web sights to look up and to comment upon. Some of these have been selected because they are very poorly done. Others have been chosen because they are well done. You will be asked to decide which and to explain the reasons for your decisions.

These exercise will be posted on the web and I will notify you of the URL when that happens. Your comments on the sights should be posted on our email list so that all can benefit from what you learn.

This exercise is due February 13. It is worth 50 points.

**Exams:** The midterm exam will be given March 27; the final exam on Friday May 8 from 3:30 to 5:30 P.M. A sample exam will be distributed prior to each exam. There will be true-false questions, matching, fill-in-the-blanks, multiple choice questions, short answer questions, and essay questions. Some questions will be closely related to the exercises at the ends of the chapters. The exams will include all of the material in the book, as well as that discussed in class.

**Research Paper:** You are to write a paper on a topic of your choice. This is meant to be an interesting and enjoyable assignment, not a chore. So choose a topic with care. The only restriction that I impose is that it cannot be on the mathematician that your first library assignment dealt with.

You should think about the choice of a topic for your paper while you are doing the two library assignments. The exercises at the ends of the chapters in Katz suggest many possible topics. Some students prefer to write about a mathematician, others prefer the history of some mathematical topic. You are encouraged to talk to me (during office hours or via email) about possible topics. As soon as you have an idea, please let me know so that I can suggest possible references or make comments about the reasonableness of your choice of topic. On February 25 I would like each of you to give me your topic via email; only rarely will I veto a topic as unreasonable. Here are some [paper topics](#) that students have chosen in the past.

Each paper must meet the following requirements:

1. The papers are to be on the history of mathematics. They can be neither all history nor all mathematics. Each should contain a reasonably non-trivial piece of mathematics as well as the history and background of that mathematics.
2. Enough expository material should be included so as to make the paper self-contained. If you have doubts, ask a friend to read it. Having someone else read your paper critically is the best way to improve the exposition.
3. You should use a variety of research materials and must give careful references to your sources. You will want to use books and encyclopedias, but I especially encourage you to use the journals (a necessity for B work). Your paper should include a bibliography listing your sources and they should be cited in the body of your paper when appropriate. The best sources to use are original sources, but, admittedly, that is hard to do. Their use is, however, required for A work.
4. The paper must be prepared using a wordprocessor (you may write in symbols if the wordprocessor you are using does not handle them); if you don't know how to use one, now is the time to learn. Other issues such as the length, format, etc., are up to you. Since you will be startled by this last comment, let me point out that papers have a natural length. You are telling a story which needs certain background, exposition, and detail. When that is successfully done, stop; you have finished. You should turn in two copies of your paper as I intend to keep one copy.

The **grading of your paper** will be based on a number of factors, including: the historical and mathematical content; the significance, interest, accuracy, and completeness of the material; the accuracy, scope and significance of your references, and the sensitivity with which they are used and cited; and finally, the style in which it is written (poorly written papers will not be accepted). As in Olympic figure skating, your score will be a combination of technical performance and artistic merit. The grade of A will be given only for truly excellent work which uses original sources; B for good solid

work that makes use of high quality journal articles; C for average work; D and F for unsatisfactory work. All grades are possible.

Here are some [suggestions](#) for writing your research papers.

To guarantee that you devote sufficient thought to your paper a topic is due February 25 and a preliminary report and outline is due March 18. The latter should include (a) your topic, (b) a few words about what you intend to do, (c) an outline, (d) your preliminary bibliography, and (e) any questions you have about your paper. This is best done via email as it makes it easier for me to incorporate comments. The more details you include, the more feedback you will get from me. Some students choose to turn in a first draft a little later. Feel free to ask questions and to indicate the problems you are having. The intent of this preliminary report is twofold: It should aid you in writing your paper and it allows me to make suggestions. Remember, the secret of good writing is rewriting.

The final version of your paper is due April 22.

**Problems:** There are many problems in the textbook and they contain a great deal of information about the history of mathematics. You should read all of them (as well as the references and footnotes for they give you some idea of the wealth of information available). Pick one problem from each chapter and do it (I encourage you not to pick the most routine ones). I will also suggest problems that you should pick from. You should do these problems in groups of cardinality three (this is called cooperative learning). Get together and work the problem, then write up a full solution with explanations. Have others in your group check and amend the write up before it is turned in. Put all three names on the paper. When the problem is turned in, it will be given to another group of three for grading. The second group will check if the problem has been done correctly, and will make comments on how good the explanation was, and will make suggestions for improvement. I will then look at the paper and make comments to both groups. Each problem is worth 15 points for doing it, and 5 for correcting it. Groups doing especially challenging problems or turning in exceptional solutions will receive bonus points. Groups can do additional problems for extra credit. You can turn in these problems when you finish with them, but must do so within 3 class days of the discussion of the chapter in class (this is to prevent everything from being turned in at the end of the semester when all of us will be too busy to deal with them effectively).

Each week I will call for volunteers (or, if needed, designate people) to put problems on the board.

**Class Attendance and Participation :** You are expected to attend class. Roll will be taken at the beginning of the semester so that I can learn your names. If you miss class you are expected to find out what happened in class from a classmate and to learn the material on your own. You are also required to present a written note (on nice paper) to me explaining your absence and apologizing. I will randomly call on people to discuss what they learned in the readings, and to suggest those topics that they are having trouble with.

**Plagiarism:** According to the Random House College Dictionary plagiarism is "the appropriation or imitation of the language, ideas, and thoughts of another author, and representation of them as one's original work." Scrupulous care must be taken to avoid this in your writing. Naturally the source of a direct quotation must be cited. But also when you take the ideas of another and rephrase them you must cite your source. In historical work everything except the common and readily available facts needs a reference to the work where you learned this information. **Mutilation of library materials is a crime, both literally and figuratively.** Xerox is cheap and readily available, so there is no excuse for defacing library holdings in any way.

Cheating of any form will not be tolerated and will be treated with the utmost severity. See your student handbook for details.

#### **SUMMARY OF DATES:**

<b>Date</b>	<b>Assignment</b>	<b>Points</b>
January 16	Automathography	25
January 30	First library assignment	50
February 6	Second library assignment	50
February 13	Internet assignment	50
February 25	Topic for paper	
March 27	Midterm Exam	100
March 18	Outline of paper	25
	Problems	200
April 22	Paper	150
	Classroom performance	50
May 8, 3:30--5:30	Final exam	150

You are expected to turn in the assignments on the dates indicated above. There will be a 10% penalty for each class that your assignment is late.



# MA376—History of Mathematics

Spring 2005

Many mathematics students go through their majors without ever realizing that mathematics has a history. Their classes focus on modern ideas treated from modern points of view, or focus on process to the exclusion of all else. They are never confronted with the fact that mathematics is a human production, socially and historically situated, nor do they get a glimpse of the subject as a whole. It is hardly possible to make up for this with one class in the history of mathematics... but we will try nevertheless.

Since mathematics has been around for at least 4000 years, no history of mathematics course can be exhaustive. In fact, many things about the history of mathematics are still unknown! What this course hopes to achieve is to give you the basic knowledge and the basic scholarly tools to allow you to go on to your own investigations. I hope that at least some of you will be sufficiently intrigued to continue to learn more.

We will begin the semester by developing a skeleton outline of the history of mathematics as a whole. Then we will go back and look carefully at one thread within that history. This semester, that thread is the evolution of calculus and analysis. Even this, of course, is far too large a story to cover exhaustively. We will be particularly interested in understanding the difference between the Ancient and the Early Modern approaches to the basic problems of the calculus, and also in the slow development of the modern notion of rigorous analysis.

It will perhaps be helpful to spell out some what I hope students will get out of the course. When we're done, I hope that each of you...

- ... will know an overall outline of the history of mathematics and will be able to place events at the proper point of the development of the subject;
- ... will be able to read original sources with insight and comprehension, be able to draw conclusions from your reading and to relate these conclusions to the overall history of the subject;
- ... will be able to locate and read the secondary literature in the history of mathematics and interact critically with it;

But aren't Kafka's Schloß and Æsop's Œuvres often naïve vis-à-vis the daemonic phoenix's official rôle in fluffy soufflés?

- ... will be able to express the result of your research and reading both orally and in written form.

Of course, I also hope that at least some of you will decide that the history of mathematics is the world's most fascinating subject and that most of you will have found the overall experience useful and satisfying.

Now to the nitty-gritty of a syllabus:

**Where to find me:** Here's the basic data

office: Mudd 407  
phone: 3278  
email: [fqgouvea@colby.edu](mailto:fqgouvea@colby.edu)

If you need to reach me when I'm not in my office, email is the best method. If you prefer, however, feel free to call and leave a message.

**Office hours:** I will be in my office and available for questions, discussion, and general conversation on Tuesdays and Fridays between 1:00 and 3:00 PM.

If you can't come during any of these times, please email or call and make an appointment. You are encouraged to come see me. It is part of your education, and one of your privileges as a Colby student.

**What will happen in the course:** This course will have three parts. First, we will read and discuss a survey of the whole history of mathematics. Second, we will study Ancient mathematics, looking carefully at samples of how these mathematicians dealt with what we now consider aspects of the calculus. Third, we will look at the Early Modern development of the calculus and the transition to a more rigorous analysis.

For the most part, classes will be lectures and discussion of the material in the reading. Reading is a huge part of this course. We won't even try to cover in class all the material I'll ask you to read. (So if there's something you want to discuss, you should say so!) But I do expect you to read and absorb this material; assessing whether you have done that is one of the things I'll be trying to do in exams and assignments.

**Texts:** We will use *Math through the Ages* as a source for a broad survey and for introductory discussions of some of the material. For the Ancient

math portion of the course we will use Euclid's *Elements* and various texts that are included in the coursepack. The remainder of the course will deal with the topics discussed in *From the Calculus to Set Theory*. As we go along, there will also be material placed on reserve and readings that are available electronically.

Of course, you will also be making extensive use of the Colby library, which has a good collection of books and reference works on the history of mathematics and the history of science. It is important to become familiar with these tools and to learn to use them. In addition, our library has a respectable rare books collection which includes several items related to the history of mathematics.

**Automathography by email:** your first assignment is to send me a short email message introducing your mathematical self to me. Tell me about courses you have taken, about ideas that you've found exciting, about things you've found boring or difficult, about your goals as a student. I'd also like to know whether you have taken any history classes at Colby. If you have worries about any special problems or needs, let me know. You are encouraged to be creative in your response; don't just answer the questions above, but include whatever else you wish. This email message is due on or before **Monday, February 7**.

**Assignments:** There will be three large assignments, which will involve reading, writing, and oral presentation. The first will be biographical, centered on the life and work of one mathematician. Specifics will be provided on the first day of class. The second will be an opportunity for you to engage with the secondary literature. The third and final one will involve studying the history of a particular topic. Specifics on the second and third assignments will be provided later.

**Final Exam:** Our final exam (*not* a take-home exam, sorry) will contain some factual material and some mathematics, but its major focus will be on your ability to synthesize materials from the course into a rich web of concepts and ideas. I will provide you with more details as we get deeper into the course.

**Attendance and Participation:** Class attendance and participation are a *required* part of this course and will influence your grade. It is your respon-

sibility to come to class having done the readings and given them serious thought and to be prepared to help the class discuss the material.

**Cheating and Plagiarism:** You are encouraged to interact with others as you do the readings (in fact, it will be a lot more fun doing it that way). Your papers, however, must be your own. You may, of course, seek help from any and all sources, but in the end what you write must be a result of your own thought processes and your assessment of the source material. Do not quote without attribution, and do not state as fact the opinions of one of your sources. Footnotes and bibliographic references are required. Feel free to discuss any questions you have about this with me. Also, please read the Colby policy on academic honesty as stated in the College Catalogue.

**Grading:** Your grade will be computed as follows:

three papers	60%
class participation	15%
final exam	25%

**A tentative schedule:** Here is the tentative schedule, with a list of readings for each class. As the course progresses, we may make changes; if so, I will let you know in advance. The readings listed next to a date are those you should have *finished* by that date. Feel free to read ahead (in fact, I strongly recommend that you do read ahead), but make careful notes so that you can review before the class discussion. I have tried to list as many of the readings as I can, but a few readings may be added as we go along. Keep reading your email.

**Feb 2** History of Mathematics: why, how, and for whom.

**Feb 4** Survey of Ancient Mathematics. *Math through the Ages*, pages 5–24.

**Feb 7** Survey of Medieval Mathematics. *Math through the Ages*, pages 25–34.

**Feb 9** Survey of Modern Mathematics. *Math through the Ages*, pages 35–60.

Most money is tainted: t'ain't yours, and t'ain't mine.

- Feb 11 Euclid. *Math through the Ages*, Sketches 14, 15, 19; *Elements*, book I.
- Feb 14 Euclid. *Elements*, book I.
- Feb 16 The Pythagorean theorem. *Math through the Ages*, Sketch 12.
- Feb 18 Incommensurability. Fowler (coursepack); Knorr, “Rational Diameters” (electronic reserve); *Elements*, skim book X, pages 235–244.
- Feb 21 Euclid’s theory of proportions. *Elements*, book V.
- Feb 23 Euclid’s theory of proportions.
- Feb 25 Euclid’s theory of proportions.
- Feb 28 Archimedes. *Quadrature of the Parabola* (coursepack); if you need help following the argument, look at the discussion in Stein, *Archimedes*.
- Mar 2 More Archimedes.
- Mar 4 Ptolemy and the table of chords. *Math through the Ages*, Sketch 18; sections from the *Almagest*; *How Ptolemy Constructed Trigonometry Tables* (both in coursepack).
- Mar 7 Wrap-up: what the Greeks knew.
- Mar 9 Issues of transmission.
- Mar 11 Mathematician talks.
- Mar 14 Mathematics in India and in medieval Islam. *Companion Encyclopedia of the History and Philosophy of Mathematics*, articles 1.6, 1.7, and 1.12 (in the reference section of Olin Library; articles by David Bressoud (on Reserve) and Victor Katz (electronic reserve)).
- Mar 16 Medieval mathematics overview.
- Mar 18 Mathematician talks.
- Spring Break
- Mar 28 Early modern mathematics. *Math through the Ages*, Sketches 11, 16, 21.

- Mar 30** Tangents. *From the Calculus through Set Theory*, 1.1–1.8
- Apr 1** Mathematician talks
- Apr 4** Quadrature. *From the Calculus through Set Theory*, 1.9–1.13.
- Apr 6** Newton and Leibniz. *From the Calculus through Set Theory*, 2.1–2.4; Leibniz’s original paper (coursepack), Rickey, *Newton: Man, Myth, and Mathematics* (electronic reserve).
- Apr 8** Mathematician talks.
- Apr 11** Leibniz’s school. *From the Calculus through Set Theory*, 2.5–2.9.
- Apr 13** Berkeley’s critique. *From the Calculus through Set Theory*, 2.10–2.12; *The Analyst* (electronic reserve).
- Apr 15** Euler. Selections from Euler (coursepack); Boyer, *The Foremost Textbook of Modern Times* (electronic reserve).
- Apr 18** More on Euler.
- Apr 20** The notion of a derivative. Grabiner, *The Changing Concept of Change* (electronic reserve).
- Apr 22** The need for analysis. *From the Calculus through Set Theory*, 3.1–3.5.
- Apr 25** Cauchy’s work. *From the Calculus through Set Theory*, 3.6–3.9, Grabiner, *Who Gave You the Epsilon?* (electronic reserve).
- Apr 27** Developing a theory of functions. *From the Calculus through Set Theory*, 3.10–3.15.
- Apr 29** Riemann and the foundations of analysis. Laugwitz, “Riemann’s Dissertation” (electronic reserve).
- May 2** Weierstrass and the foundations of analysis.
- May 4** Modern theories of integration. *From the Calculus through Set Theory*, 4.1–4.7.
- May 6** Analysis in the 20th century.

Syllabus for HM/MA 004, “Calculus and its History”  
Semester I 2002/03

*Readings, assignments, and syllabus updates are available on-line at the course homepage:*  
[http://www.brown.edu/Departments/History\\_Mathematics/HM0004/hm4.html](http://www.brown.edu/Departments/History_Mathematics/HM0004/hm4.html)

Instructor:	Kim Plofker
Department:	History of Mathematics
Email:	Kim_Plofker@Brown.edu
Office:	Wilbour Hall, Room 001
Office phone:	863-1489
Office hours:	W 2:30–3:30, Th 11:15–12:15, 3:30–5:00

**Course Overview:** Currently offered as a first-year seminar, “Calculus and its History” is intended for students (whether or not they have already studied calculus) who would like to investigate questions like the following:

- What is calculus? Who invented it?
- When and how did it develop?
- Why is it harder than the math I know already? (In what ways is it easier??)
- What problems inspired its creation?
- In what ways did its historical setting change its development, and how did its development affect history in general?
- How did it change the way mathematicians and other people think about mathematical knowledge?

Readings of original sources in English translation range from Babylonian mathematical tablets through Euclid and Archimedes, Oresme, Galileo, Leibniz and Newton, to Cauchy, Riemann and Robinson.

The course will meet in C hour (MWF 10:00–10:50) in Sayles 204 according to the University Calendar from 4 September to 9 December 2002.

Assigned texts will consist of books, handouts, and electronic texts containing excerpts from primary sources in English translation, including (but not limited to) the following:

- *The History of Mathematics: a Reader*, by Fauvel and Gray (at the Bookstore)
- *A Concise History of Mathematics*, by D. J. Struik (at the Bookstore)
- *Mathematical Expeditions*, by Laubenbacher and Pengelley (at the Bookstore)
- The *Elements* of Euclid (<http://aleph0.clarku.edu/~djoyce/java/elements/toc.html>)
- The *Physics* of Aristotle (<http://classics.mit.edu/Aristotle/physics.html>)

There will be readings and “mini-assignments” for every class (with few exceptions) and graded homework assignments every week (with few exceptions). (“Mini-assignments” are brief, one-sentence/one-paragraph written responses to questions about the readings. They are not graded or returned like the weekly homework assignments, but submitting them counts toward the class-participation part of the course grade.) There will be a 7–10 page midterm essay due 18 October, and a 13–15-page final research paper due 18 December.

Determination of course grade:

Class participation and mini-assignments	20%
9 graded homework assignments (lowest two of 11 grades dropped)	30%
Midterm essay	20%
Final paper	30%

Date	Topics
W 9/4	<b>Introduction.</b> (What is calculus? How do we approach it?) Discussion of pre-calculus mathematics, students’ mathematical backgrounds. Reading original mathematics: Babylonian tablet. Explanation of course requirements, course objectives. Focus on original sources, historical development, and mathematical understanding.
F 9/6	<b>Before Calculus: Practical mathematics in ancient civilizations.</b> (We explore the basic mathematical tools that ancient cultures developed for problem-solving, and link them to some of our modern tools such as notation.) “Original urban-civilization” mathematics: Egyptian/Babylonian arithmetic. Babylonian area calculations. (Explicit discussion of algebraic “translation” of pre-algebraic works.)
M 9/9	<b>Philosophical limitations to mathematical knowledge.</b> (We discuss the evolving definition of mathematical knowledge and its shocking consequence: the discovery that we can know exactly that there are things we cannot know exactly.) Babylonian into Greek mathematics. The role of mathematics in Greek philosophy. What is number? The Pythagoreans: numerical cosmology, ratios, means. Commensurability and irrationality.
W 9/11	<b>Developing mathematics within the limitations, I.</b> (The ancient Greeks ask: If we concede that there are limits to what mathematical knowledge can tell us, what can we know within those limits?) How do we separate what is mathematics from what isn’t? The axiomatic deductive method and the geometrical approach: strict definitions of number, separation of number from magnitude. Geometric proofs and their “plots”.
F 9/13	<b>Developing mathematics, II.</b> (What kind of problems can Greek mathematics solve and still be confident that it produces exact knowledge?) Difficulties with intuitive notions of infinity and continuity. Zeno and the paradoxes of motion.
M 9/16	<b>Areas and volumes.</b> (In the days before coordinates and graphs, how do we talk about curved lines? Can we discuss any of them mathematically? Which ones? How?) Quadratures and cubatures. Squaring rectilinear figures; squaring the lune.
W 9/18	<b>Archimedes; curves other than the circle.</b> (The greatest mathematician of antiquity comes up with some clever answers.) The method of exhaustion and reduction to absurdity; “principle of Eudoxus.” The quadrature of the parabola.
F 9/20	<b>The “Method” for finding solutions.</b> Mechanical arguments for computing with “infinitesimal” areas and volumes. Indivisibles and the quadrature of the parabola. “Cleaning up” the results with rigorous proof.
M 9/23	<b>The end of classical mathematics; the “other” classical mathematics.</b> (What ever happened to Greek mathematics? What did the rest of it look like?) The declining impact of classical geometry in late antiquity. Efforts to systematize and classify it; ideas of analysis and synthesis. The other side of Greek mathematics: the need for computation.
W 9/25	<b>The “other” classical mathematics, cont’d.</b> Astronomical/geographical problems: trigonometry, computing chord values. Classical systems for representing and calculating with numbers.
F 9/27	<b>Mathematics in motion, I; new tools from other cultures.</b> (Drawing pictures of change: late medieval mathematicians try to quantify change and motion.) Discussion of the latitude of forms. Qualities and quantities; rates of change; the mean speed theorem. Calculation techniques and the “Arabic” numbers.
M 9/30	<b>New tools, II: the emergence of early modern math.</b> Islamic algebra techniques. Picking up the separate strands of “original-urban-civilization” math, classical Greek geometry, the “other” classical mathematics, Indian numerals, Islamic developments, the European university curriculum: unification into the Latin mathematics of the early modern period, with its “analytical art”.
W 10/2	<b>New people and new ideas in mathematics.</b> Innovative applications and developments attract popular interest in mathematics. “A very large number of very small pieces:” new approaches to calculating areas and volumes. Methods of indivisibles, Cavalieri’s principle. Philosophical objections. The integral power law.
F 10/4	<b>New tools, III; computational requirements spurring mathematical developments.</b> Me-



dieval development of trigonometric functions from ancient Greek chords. Trigonometric tables and problems, and their computational burden. Number-crunching with Logarithms: replacing multiplication by addition.

- M 10/7 **Mathematics in motion, II.** (Mathematicians return to the discussion of motion and change, and draw more pictures.) Classical and medieval notions of motion: quantifying celestial motions. Quantifying terrestrial motion: velocity and acceleration, projectiles, mathematical and graphical descriptions of motion in Galileo.
- W 10/9 **The beginning of general solutions, I.** (The amazing discovery that a lot of problems that look different are really the same.) Descartes: analytic geometry and coordinate systems. Introduction of equations for curves. The principle of nonhomogeneity. Fermat’s  $e$  and the “adequality” approach for maxima and minima.
- F 10/11 **Beginning of general solutions, II.** Review of conic sections in ancient and early modern guises. Representing the curve and finding the tangent. Barrow: finding tangents can be related to finding areas.
- M 10/14 NO CLASS
- W 10/16 **(Mid-semester) Systematizing the math of arbitrarily small variations.** Review/preview of course topics week by week. Identifying the two basic types of problems for these methods: tangents/maxmins, areas/volumes.
- F 10/18 **General solutions, III: the techniques for attacking the two basic types of problems.** Sample methods for tangents/maxmins: Fermat’s dividing by not-quite-zero. Sample methods for areas/volumes: unexpected application to finding line-lengths.
- M 10/21 **Historical context of the calculus, I.** (Why is all of this mathematics happening now? How are these concepts justified?) Historical and philosophical background of the calculus in the early seventeenth century: education, technology, motivation.  
(The “Scientific Revolution” as a historical concept, 1450–1750. Movable type and the fall of Constantinople; implications for revival of ancient learning. Factors fostering the redevelopment of research mathematics: intermediate social class(es); educational institutions and resources; employment opportunities; source of research problems; collegial communication. Comparison of medieval universities’ curricula with actual influences on 16th–17th c. mathematicians’ training and employment. Element of chance in mathematical careers; absence of real professionalization of the field at this point. Philosophical/mystical acceptance of paradox permits more speculation on notions such as the infinitely large and small.)
- W 10/23 **The differential calculus, I: Leibniz.** (At last, a truly general algorithm for all (?) curves.) Pascal’s sines and the “tiny tangent triangle.” Leibniz’ *New Method* and basic rules of differentiation.
- F 10/25 **Class visit to Lownes Collection of John Hay Library.** View early editions of Euclid, Fermat, Descartes, Leibniz, Newton, etc., as well as some of Brown’s cuneiform text collection. Discussion of the physical sources of mathematical texts, their dissemination and use.
- M 10/28 **The differential calculus, II: Newton.** (Simultaneously (?), another truly general algorithm.) Completing Leibniz’s “calculus of differentials” with the equivalent of the chain rule. Comparing this approach to Newton’s  $o$ -method and fluxions: motivation from physical problems of motion.
- W 10/30 **Historical context, II: the great dispute.** (Who invented the calculus? Why are we fighting about it?) Reception of the two calculus techniques and the argument over priority and plagiarism. National loyalties (the “scurvy English” vs. the “Leipzig rogues”); the Royal Society’s inquiry.  
(“Protocalculus” vs. “calculus” concepts: justification of attribution of calculus to Newton and Leibniz rather than Fermat or Barrow. Power and flexibility of the new techniques; proliferation of calculus textbooks; fame and prestige of the inventors. Chronology of the discoveries and the accusations in the dispute; challenges and counter-challenges. Extracurricular motivations: political issues concerning England and Hanover; Newtonian vs. Continental physics.)
- F 11/1 **The differential calculus, III: further testing.** (Does the new “calculus of differentials” work for all the curves we can think of?) Interpretation of Bernoulli and L’Hôpital; systematization into

early calculus books. Profusion of applications to physical problems. Extrema, concavity, higher derivatives.

- M 11/4 **The integral calculus, I: Leibniz and Newton again.** (The two inventors present their versions of the other half of the calculus.) Leibniz and Newton on integration; using the fundamental theorem to find areas.
- W 11/6 **The integral calculus, II.** (Systematizing the techniques of integration.) Finding the area under various curves; tables of integrals. Rectification.
- F 11/8 **Historical context, III.** Making an algorithm into a discipline: notation, textbooks, inclusion in the curriculum, new problems.  
(Late 17th–late 18th c.: calculus changes from research mathematics to instruction for schoolchildren and even suggested recreation for ladies. How has it been “mainstreamed”? Development of standardized mathematical notation; Sudden development of mathematical “alphabet” in 17th c. to express expanding range of concepts and procedures. Mathematization at present affects only the *physical* sciences; example of a contemporary calculus textbook and its “physico-mathematical” problems.)
- M 11/11 **The integral calculus, III.** Further techniques of integration and other applications. Tying integration in with differentiation.
- W 11/13 **Historical context, IV: philosophical objections to the calculus.** (“The emperor has no clothes, and he’s dividing by zero.” complaints that the “new analysis” has abandoned mathematical certainty.) Newton and Berkeley. Fluxions and the “ghosts of departed quantities.”  
(The Academy of Lagado in *Gulliver’s Travels*: satirical assessment of the Royal Society and “progressive” attitudes toward science. “Responsible opposing viewpoint” re the Scientific Revolution, shared by Berkeley. Developments in sciences and scholarship color Enlightenment ideas of literal truth in religion. The *Analyst* as a response to freethinking abetted by mathematical rationalism. Details of criticisms: higher fluxions and differences inconceivable, manipulation of infinitesimals illogical.)
- F 11/15 **Responses to the critiques.** (Discussion of final paper: study a renowned problem solved (or controversy caused) by calculus and write a 13–15-page paper explaining the mathematics and discussing its historical development. A list of sample topics and a description of desired goals and methods for the paper will be provided.) Counterarguments provided by the reasoning of Leibniz, Nieuwentijdt, Bernoulli, Grandi, Euler. Is the calculus philosophically defensible? Need we care?
- M 11/18 **The calculus explosion, I: Euler.** (Is there anything calculus can’t solve? Eighteenth-century mathematics takes off.) The differential equation; power series. Leonhard Euler and the new organization of mathematics.
- W 11/20 **The calculus explosion, II.** The exponential and the logarithm.
- F 11/22 **The calculus explosion, III.** (Is there any physical problem calculus doesn’t apply to?) Euler and the world of “mixed mathematics”.
- M 11/25 **Rigorization of the calculus, I.** (Can we restore mathematical certainty to the calculus? Sure; just change the definitions.) Educational necessities lead to a re-examination of philosophical complaints about the calculus. D’Alembert, Lagrange, Cauchy and the limit; definition of the derivative.
- W 11/27 **NO CLASS**
- F 11/29 **NO CLASS**
- M 12/2 **Rigorization, II.** (More changing of definitions.) Definite integrals: Cauchy and Riemann. Is this just Archimedes all over again?
- W 12/4 **Rigorization, III.** (The dreaded epsilon-delta limit finally appears.) Foundations of the limit: Heine and Weierstrass, epsilon-delta definition. The “fundamental theorem.”
- F 12/6 **Conceptual changes after the 19th century.** (How did all this become the calculus we know today? Is it going to stay that way?) Standard approach to the rigorous calculus. Nonstandard analysis; Robinson and infinitesimals.

M 12/9    **Conclusion.**

**Math 108: History of Mathematics**

**Fall, 2003**

**Prof. J. Grabiner**

**Office phone: 73160; Secretary: 621-8218; email  
jgrabiner@pitzer.edu Office Fletcher 224; Office Hours posted**

**Required Books:**

Victor J. Katz, A History of Mathematics: An Introduction (2d ed., Harper Collins, 1998)

(This is the best one-volume history in existence, and it is worth buying, and keeping, even at the "new" price.)

Plato, Meno (Bobbs-Merrill pb)

René Descartes, Discourse on Method (Bobbs-Merrill pb)

Jacques Hadamard, Psychology of Invention in the Mathematical Field (Dover pb)

**Week Topic and Assignments** (Lectures will focus on specific topics, and will reflect the instructor's view of what is most important; Katz will provide an overview and some details.)

Sept. 3 **Introduction to the course**

Sept. 8, 10

**Mathematics in Egypt and Babylon; Mathematics in Africa, the Americas, & the Pacific.**

Katz, 1-45, 332-341.

Recommended: H. Frankfort, Before Philosophy; O Neugebauer, Exact Sciences in Antiquity

M. Ascher, Ethnomathematics; Carl Boyer and Uta Merzbach, A History of Mathematics; B. L. van der Waerden, Science Awakening; A. Aaboe, Episodes from Early Mathematics.

Sept. 15, 17 **From Pythagoras to Euclid.**

Katz, 46-58

Plato, Meno (all) for Wed. discussion

Recommended: W. Knorr, The Evolution of the Euclidean Elements; W. Knorr, The Ancient Tradition of Geometric Problems; David Fowler, The Mathematics of Plato's Academy.

Sept. 22, 24 **Euclid and His Time**

Katz, 58-101;

Recommended: Aaboe, op. cit., and (superb!) Knorr, The Evolution of the Euclidean Elements; T. L. Heath, ed., The Thirteen Books of Euclid's Elements (Euclid, in full and in English).

Feb. 29, Oct. 1 **Archimedes**

Katz, 102-134. (See more assigned reading 5 lines down)

Recommended: T. L. Heath, ed., The Works of Archimedes; E. J. Dijksterhuis, Archimedes (2d edition has updated bibliography)

**Apollonius, Diophantus, Hellenistic Mathematics**

Katz, 135-191.

Recommended: J. Klein, Greek Mathematical Thought and the Origin of Algebra

Oct. 6: No Class (Yom Kippur)

Oct. 8, 13 **China and India; Mathematics in the Islamic World**  
Katz, 192-237; 238-287.

Recommended: George Gheverghese Joseph, The Crest of the Peacock: Non-European Roots of Mathematics; Frank J. Swetz and T. Kao, Was Pythagoras Chinese? Right Triangle Theory in China; J. L. Berggren, Episodes in the Mathematics of Medieval Islam, N. L. Rabinovitz, Probability and Statistical Inference in Ancient and Medieval Jewish Literature

Oct. 15 **The Latin Middle Ages**

Katz, 288-326; 327-331.

Recommended: M. Mahoney, "Mathematics," and J. Murdoch and E. Sylla, "The Science of Motion," in D. Lindberg, ed., Science in the Middle Ages

Oct. 20 Fall Break

Oct. 22 **The Renaissance, and the Scientific Revolution.**

Katz, 342-384, 385-430. Also Katz 544-595.

Recommended: O. Ore, Cardan: The Gambling Scholar; J. Klein, Greek Mathematical Thought and the Origin of Algebra; Judith Grabiner, "Mathematics," in P. F. Grendler, ed., Encyclopedia of the Renaissance (6 vols., 1999), vol. 4, 66-72. On the Scientific Revolution, recommended: I. B. Cohen, The Newtonian Revolution

Oct. 27, 29. **Analytic Geometry**

Katz, 431-448, 448-503.

Descartes, Discourse on Method, all. (For discussion Monday, October 27)

Recommended: M. Mahoney, The Mathematical Career of Pierre de Fermat. C. Boyer, History of Analytic Geometry

Nov. 3, 5. **17th-century mathematics before the calculus; Newton and Leibniz**

Katz, 448-467; 468-503. Then, 503-543.

Recommended: I. Hacking, The Emergence of Probability; C. Boyer, History of the Calculus (covers antiquity - 19th c.) M. Baron, Origins of the Infinitesimal Calculus (focus on 17th c.) A. R. Hall, Philosophers at War: The Newton-Leibniz Controversy

Nov. 10, 12 **18th-century mathematics**

Katz, 544-595, 596-648.

Recommended: T. L. Hankins, Science and the Enlightenment; M. R. Kline, Mathematical Thought from Ancient to Modern Times (relevant chapters); H. Goldstine, History of Numerical Analysis from the 16th through the 19th century

Nov. 17 **Rigorization of the calculus.**

Katz, 704-737. Handouts: translations from Cauchy.

Recommended: J. V. Grabiner, The Origins of Cauchy's Rigorous Calculus

Nov. 19 **Mathematical creation** (class discussion Tues.) based on Hadamard, Psychology of Invention in the Mathematical Field

Nov. 24: Film, "The Proof" (on Fermat's Last Theorem) and

discussion

Nov. 26: to be announced

Dec. 1, 3: **Women in Mathematics:** Student Reports in Class.

Details and suggested sources will be forthcoming. Each student will have  $k$  minutes where  $k + 1 = \frac{\text{total time}}{\text{number of students}}$  for the in-class report. Students not coming to present an in-class report will be required instead to hand in a documented 3-page written version, due Monday December 8 at the start of class.

Recommended: J. Alexanderson & D. Albers, Mathematical People; More Mathematical People; Claudia Henrion, Women in Mathematics (her bibliography is especially worth consulting); C. C. Gillispie, ed., Dictionary of Scientific Biography, 16 vols., the place to start for reliable biographies of any deceased scientific figure; Ann H. Koblitz, Sofia Kovalevskaya: Scientist, Writer, Revolutionary; Constance Reid, Julia Robinson.

Dec. 8, 10: **Modern Mathematics; Bringing it All Together**

Katz, (skim) chapters 15, 16, 17, 18. Choose **one** section based on your own mathematical background.

Recommended: T. Hawkins, Lebesgue's Theory of Integration; J. L. Richards, Mathematical Visions: Non-Euclidean Geometry in Britain; G. Moore, Zermelo's Axiom of Choice; Constance Reid, Hilbert, Sylvia Nasar, A Beautiful Mind (life of John Nash)

## Written assignments and grading:

### (i) Daily assignments:

At the end of each class, hand in one legible sheet of paper with (1) the key point of the day and (2) one question raised by the day's class.

(ii) Weekly assignments: Every Monday (except Sept. 8, the first real week, and Dec. 1, the week reports are given), you will hand in the following:

- (a) the solution of any **problem** in the previous week's chapter from Katz (your choice; it's most valuable if you pick the hardest one you can reasonably do);
- (b) your answer to **one** of the **discussion questions** in the previous week's chapter(s) from Katz (again, your choice; pick something that interests you. *Do not "share" this assignment with another student; work on your own*). **One page, double-spaced**, should be enough for each question.
- (c) **either** an outline of the material in the previous week's chapter(s) in Katz, **or** half a dozen intelligent sentences, in your own words, about the six individual topics in those chapter(s) that you found interesting. But if you do the six topics, they must not duplicate either the problem or the question you do from this chapter.

For both (a) and (b), GIVE THE PAGE AND QUESTION NUMBER!  
**Please type (b) and, if possible, (c); make (a) as legible as possible.**

(iii) Report: Each student will do a brief report the week of December 1-3. See calendar. More details will be provided closer to the date.

(iv) **Final Examination** (short answer plus essay): Tuesday, December 16, 2 PM. Study suggestions will be provided.

**Grading algorithm**: Weekly assignments 45%, daily assignments 15%, report 15%, final exam 25%.

**Late work (without compelling reason) docked 10% per CALENDAR day.**

**Mathematics 504**  
**History of Mathematics**  
**Spring Quarter 2005**  
**Call number 12231-2                      MWF 3:30 – 4:50 pm**  
**Baker Systems Engineering BE192**

**Bostwick Wyman**  
**Room MW434 Math Tower**  
[Wyman.1@osu.edu](mailto:Wyman.1@osu.edu)  
**292-4901**

**Office Hours: Daily, 1:30 to 3:00 pm.**

**Prerequisites:** Math 568 or Math 580 or Math 547 or permission of department. Math 507 will be very helpful. On the other hand, this course will be very helpful for Math 507, so either order is okay. Math 345 will also be very helpful.

**Course Philosophy:** This course will emphasize the historical context of mathematics. A major term paper as well as an oral presentation and a shorter book review and a brief biography project will be required. See below for details. If a third writing course were to be required in the Mathematics Department, this would be one.

**Text and Resources:**

*Math Through the Ages: A Gentle History for Teachers and Others; Expanded Edition.*  
William P. Berlinghoff and Fernando Q. Gouvea, Oxtan House and M.A.A., 2004.

Extensive use of History of Mathematics web sites

Notes by the instructor and various handouts

**Course Requirements and Grading:**

Midterm	Monday May 2	10%
Book Review		10%
Brief Biography of a “recent” mathematician		10%
Critique of Draft Paper by another student		10%
Term Paper (including abstract ,first draft, talk)		40%
Homework, quizzes, attendance		20%
There will be no final examination.		

**Regular attendance is expected.** There will probably be a short quiz or written work due in class every day. Only serious illness or university business should prevent you from attending class. If you have a foreseeable conflict, I will be a *little* more sympathetic if you tell me ahead of time.



## Time Table

**Monday, April 11:** Deadline for choice of topics for Book Review, Biography, and Term Paper. See lists for suggestions of books and paper topics. Biography must be of a mathematician born after 1899. Your Book Review, Biography, and Term Paper must have *different topics!* You might want to choose your topics soon, since no two students can work on the same book or the same person or the same paper topic. Be sure to check with me before you start working on your projects.

**Monday April 18:** Book Review due, hard copy. I will read it quickly and return it with comments. Electronic revised copy due May 4. Limit two pages. Give a brief description of the contents of your chosen book, the intended audience, and the main goal of the author. Be sure to include your opinion of the book. Your goal is to help your colleagues decide if they want to read it.

**Monday, April 25.** Biography due, hard copy. Minimum 2 pages, maximum 4. Subject must have been born after 1899. This is a brief professional biography, covering education, employment, main contributions, effect her/his work had on mathematics in general. (Your subject for the biography cannot be in your book or your term paper.)

**Friday, April 29.** Submit TWO hard copies of abstract and bibliography. Give a outline of the main ideas of your paper together with a bibliography of your source materials and references. Be sure to keep a copy! I will return the abstract and bib with comments as soon as I can.

**Monday, May 2: Midterm.** Mostly short answer and identification, a little math.

**Wednesday, May 4.** Revised Book Review due, electronic copy. Word doc or .pdf file.

**Monday, May 9:** Submit TWO hard copies of a complete draft of your paper. Drafts will be read and critiqued by the instructor and by one other student in the class (chosen at random).

**Wednesday, May 11.** Revised Biography due, electronic copy. Word doc or .pdf file.

**Friday, May 13:** Give student critiques to Wyman.

**Monday, May 16:** Drafts returned with comments and student critiques.

**May 20,23,25,27, June 1 and 3:** 10 minute oral presentations of paper, with discussion. Stay tuned for sign-up instructions. (Tentative -- more days may need to be scheduled)

**Monday, May 30.** Memorial Day, no class.

**Tuesday, May 31.** (Not a class day). Final draft of Term Paper due in electronic form. Email to Wyman or bring it on a CD-ROM. **Hard copy due in class Wednesday June 1.**

**Mathematics 504**  
**Spring Quarter 2005**  
**Calendar and Deadlines**

M	3/28	First Day
W	4/1	Read B&G 1-27
F	4/3	Read B&G 28-60
M	4/4	
W	4/6	
F	4/8	
M	4/11	Last day to choose Book Review, Biography, Paper Topic
W	4/13	
F	4/15	
M	4/18	Book Review due (hard copy)
W	4/20	
F	4/22	
M	4/25	Biography due
W	4/27	
F	4/29	Abstract and Bibliography for long paper due (hard copy)
M	5/2	Midterm
W	5/4	Revised Book Review due (electronic copy)
F	5/6	
M	5/9	First Draft of Long Paper Due (two hard copies)
W	5/11	Revised Biography due (electronic copy)
F	5/13	Comments on colleague's first draft due (to Wyman)
M	5/16	Comments on first draft returned to authors
W	5/18	
F	5/20	Talks
M	5/23	Talks
W	5/25	Talks
F	5/27	Talks
M	5/30	Memorial Day, no class
T	5/31	Final Draft of Long Paper (Electronic Copy) (email or CDROM)
W	6/1	Talks
F	6/3	Talks

## Mathematics 504 Book List

Here is a (very incomplete) list of books which might be suitable for your book review project. I am trying only to make some suggestions to give you a starting point, not to limit your choice. I have tried to choose books which are not technically difficult, and I hope I have succeeded. Remember that you have to pick a book soon. Be sure to clear your choice with me, since I only want one of you to write about a given book.

You should also look at the *Notices* of the American Mathematics Society, which has a book list in every issue. The “Telegraphic Reviews” section of the *American Mathematics Monthly* is valuable. Look also at *Mathematics Magazine*. If you google any of these authors or titles, you might find alternative possibilities. You will probably also find book reviews. These are helpful – but be careful to avoid inadvertent plagiarization or absorption of somebody else’s ideas. (Plagiarization on purpose is grounds for an academic misconduct case. Don’t do it.)

Artman, Benno, *Euclid, The Creation of Mathematics*. Springer-Verlag, 1999.

Ayoub, Raymond, editor. *Musings of the Masters*, Math Assoc. of America 2004. Essays by famous mathematicians.

Dunham, William. Many good books! *Journey through Genius*, *Calculus Gallery*, etc.

Fefferman, Anita and Solomon, *Tarski*, Cambridge Univ. Press, 2004. Biography of a famous logician.

Ferguson, *Tycho and Kepler*, Walker and Co. 2004. Astronomy and Math.

Gingerich, Owen, *The Book Nobody Read*, Walker and Co., 2004. Copernicus.

Gleick, James, *Newton*. Heritage Books, 2003. Biography. Also by Gleick: *Chaos*.

Abbot, Edwin, *Flatland*. Probably lots of editions of this old, famous book. Revised edition published in 1884. Some recent copycat and updated stuff is around.

Aczel, Amir. *The Mystery of the Aleph*. Four Walls/Eight Windows, 2000. George Cantor, set theory, and infinity.

Banchoff, Thomas. *Beyond the Third Dimension*. Scientific American, 1990 and 1996. Great pictures and good math.

Cole, K.C. , *The Universe and the Teacup*. Abacus Books. 1999. Cosmology.

Davis, Philip and R. Hersh, *The Mathematical Experience* (1981) and *Descartes’ Dream* (1986). Famous books.

Davis, Philip. *The Thread, A Mathematical Yarn*. Harcourt, 1983,1989. Lightweight but fun.

Derbyshire, John. *Prime Obsession*. Plume/Penguin, 2003. Prime numbers and the Riemann Hypothesis. There is a similar book by Karl Sabbagh and maybe a third, but I forget.

Devlin, Keith. An NPR commentator who has written lots of books. They are all good. *The Math Gene*. (2001) , *The Millenium Problems* (2002) Etc etc.

Dewdney, A.K. Lots of books . Google to web site.

Galison, Peter, *Einstein's Clocks, Poincare's Maps: Empires of Time*, Norton 2003. Recently reviewed in the New York Times.

Gamow, George and Russell Stannard, *The New World of Mr. Tompkins*. Cambridge, 1999. Updated version of a famous book. Physics, mostly relativity and quantum mechanics. Fun to read. Gamow's most famous book (an old one) is One, Two, Three ... Infinity.

Gardner, Martin. Former columnist for Scientific American. Many books, all good. Often independent chapters.

Guedj, Denis. *The Parrot's Theorem*. Thomas Dunne, 2000. Fiction with embedded mathematics.

Hersh, Reuben. *What is Mathematics Really?* Oxford, 1997. Philosophy of Mathematics. If you do a search on p. of m. you will get many more standard books.

Hildebrandt, Stefan and A. Tromba, *The Parsimonious Universe*, Springer, 1996. **Math** in Nature.

Hoffman, Paul. *The Man Who Loved Only Numbers*. Hyperion, 1998. Bio of Paul Erdős. (Pronounced: "Air-dish") Also, *My Brain is Open*, by Bruce Schechter.

Jones, Brian. Two current books about mathematics and physics: *The Elegant Universe* and *The Fabric of the Cosmos*.

Katz, Victor, Editor. *Using History to Teach Mathematics: An International Perspective* (MAA Notes # 51), Mathematical Association of America, 2000.

King, Jerry. *The Art of Mathematics*. Fawcett, 1992. Esthetics of Mathematical proof.

Lakoff, George and R. Nuñez, *Where Mathematics Comes From*. Basic Books, 2000. Linguistics and philosophy.

Livio, Mario. *The Golden Ratio*. Broadway/Random House, 2002. A famous number: architecture, nature, Fibonacci numbers . . . .

Maor, Eli. Excellent stuff: *Trigonometric Delights*. And *E, the Story of a Number*. Or any other book by Eli Maor.

Mazur, Barry. *Imagining Numbers, Especially the Square Root of minus Fifteen*. Farrar, 2003. (This was the text for a freshman honors seminar.)

Mazur, Joseph, *Euclid in the Rain Forest*. Pi Press, 2005. Logic and Mathematics.

Nasar, Sylvia. *A Beautiful Mind*. Simon&Schuster, 1998. John Nash. Not much like the movie. There is also a more technical book, *The Essential John Nash*.

Osserman, Robert. *Poetry of the Universe*. Doubleday, 1995. Mathematical Cosmology. Osserman wrote an article in the current AMS Notices. Cosmology is the topic of this years “Mathematics Month” and the annual math department honors day.

Peterson, Ivars. *The Mathematical Tourist*. Freeman, 1998. Or anything else by Peterson.

Petsinis, Tom. *The French Mathematician*. Walker and Company, 1998 Fictionalized Bio of Galois.

Salsburg, David. *The Lady Tasting Tea*. Freeman, 2001. Historical anecdotes about statistics.

Seife, Charles. *Zero, the Biography of a Dangerous Idea*. Penguin, 2000. Zero was hard to discover (invent?)

Singh, Simon. *Fermat’s Enigma*. Probably the best non-technical book about Wiles’ proof of Fermat. Bantam, 1998.

Smullyan, Raymond. *What is the Name of This Book*. Prentice Hall, 1978. The first of many wonderful logic and puzzle books by Smullyan. Any of them is okay!

Sobel, Dava, *Longitude*. Penguin 1995. To measure longitude, you need a good clock. Also by Dava Sobel: *Galileo’s Daughter*.

Tobias, Sheila. *Overcoming Math Anxiety*. Norton, 1993. (second edition) Probably this book invented the phrase “math anxiety.”

Taylor, Andrew, *The World of Gerard Mercator*, Walker and Co., 2004. Maps and the history of science during the inquisition.

Wallace, David Foster, *Everything and More*, Weidenfeld and Nicholson 2003. Recently in paper. Kinds of infinity.

## **Mathematics 504 — Spring 2005**

### **History of Mathematics —Term Paper Topics**

The timetable for the term paper is on the syllabus, which you received the first day of class. Please note that the timetable is quite strict.

The **topics** listed below are approved topics. (Many of them are due to Professors Wyman, Solomon, and Brown of the Department of Mathematics at the Ohio State University.) The goal here is to propose topics, not to limit your choices. You may propose your own topic for approval if you wish. Please keep in mind that a narrow focus usually produces a more successful paper than a broad topic. Avoid writing about “God, Humanity, and the Universe.” This injunction applies not only to new proposals but also to the official topics below, many of which may be stated too broadly. Keep in mind that the audience for your papers is your fellow class members.

**How long, oh, how long?** You should take the space to say what you have to say. Lengthy papers are fine so long as what is in them is necessary. (Padding is never necessary.) Many paper topics will require you to provide proofs of mathematical results, and for those also, use the space that you need. Remember that your proofs are written for class members, not for research mathematicians. Your papers will be judged on how well they cover their topics. While no minimum length is set, it is difficult to see how one could cover very much in less than ten pages.

The **bibliography** is important. In it you need to list sources that you are using and important references. One purpose of the bibliography is to provide a reading list of materials that would be helpful in learning more about your topic. Another important and scholarly use of your bibliography is to permit a reader to verify independently your use of source materials.

Please be very careful when citing web pages in your bibliography. Web pages are useful and easy to find (go Googling), but they are not necessarily as reliable as books and research papers, which are edited or refereed. Also, web pages can change without notice. Please print out and keep a hard copy of any web-based information you use. It is not necessary to attach this stuff to your paper, but in case you need to refer to it later, remember that it may disappear from the web.

**Preparation of the papers.** The long papers, and also the book reviews and the biographies, must be prepared on a word processor. Some drafts (and sometimes one copy of the final draft) must be submitted on hard copy so that I can make comments in the margin and return them to you. The absolutely final public drafts must be submitted electronically. I recommend .pdf files, but I will accept Microsoft word docs. It might happen that you need to include complicated formulas and diagrams which are hard to type without special software. In that case, we might have to scan in an appendix. Please see me about this later on. At the end of the quarter, I will gather up everyone’s work and burn an archival CD for each student.

## **Math 504**

### **List of Topics**

#### **Historical emphasis**

1. ***Post-1965 discoveries*** about the history of Chinese mathematics before the 17<sup>th</sup> Century. (Some standard material from textbooks might be included, since we will not cover China in the class, but emphasis should be on recent research in the history of math.)
2. Same as 1. but for Egyptian mathematics.
3. Same as 1. but for Babylonian mathematics.
4. Same as 1. but for Greek mathematics. (Careful, we will do a lot in the class about the Greeks, so new research is crucial.)
5. Same as 1. but for Indian mathematics
6. Same as 1. but for Arab mathematics.
7. Same as 1. but for another culture or civilization. This topic will need approval by the instructor.
8. The interaction of military history with the development of mathematics. Pick a specific time period for this one. (Applies also to 9 – 12)
9. The interaction of navigation with the development of mathematics.
10. The interaction of astronomy with the development of mathematics.
11. The interaction of physics with the development of mathematics.
12. The interaction of commercial activity with the development of mathematics.

#### **Philosophical emphasis**

1. Plato and his Academy and his philosophy, with special attention to the Platonic or “idealistic” approach to problem of existence of mathematical objects. (see Reuben Hersh, *What is Mathematics, Really ?*)
2. Bishop Berkeley’s philosophy and his critique of the calculus and the responses to his critique.

3. Immanuel Kant and his philosophy and his views on mathematics, geometry, in particular. Discuss any influence that it might have had on the development of nonEuclidean geometry and Gauss' reluctance to publish his findings thereon.
4. The philosophy of Ludwig Wittgenstein, and, in particular, his views on the foundations of mathematics and his critique of Bertrand Russell.
5. Kurt Gödel and the philosophical implications of his incompleteness theorem for mathematics and human thought in general.
6. Discuss current controversies concerning the death of proof and/or the stagnation of science.

**“Biographical”** In this course (Spring Quarter 2005) one of your assignments is to write a short biography of a mathematician. If you choose a long paper topic which is also biographical, please concentrate on mathematical ideas or the historical context that influenced the mathematics or the research environment of the time. I am not looking for a paper which is mostly biography, so please be careful. Also, your long paper topic must be disjoint from your short biography project! You may find it helpful to focus on only part of your subject's life and work, if appropriate.)

1. The life of Archimedes, his principal mathematical AND scientific accomplishments, his predecessors, and his influence on the future of mathematics and science.
2. The life of Galileo, his principal mathematical AND scientific accomplishments, his predecessors, and his influence on the future of mathematics and science.
3. The life of Kepler, his principal mathematical AND scientific accomplishments, his predecessors, and his influence on the future of mathematics and science.
4. The life of Euler, his principal mathematical accomplishments, his predecessors, and his influence on the future of mathematics.
5. The academic and intellectual life in England in the 17<sup>th</sup> Century, and, in particular Newton, his predecessors, rivals, accomplishments, and legacy.
6. The life and philosophy of Pascal, his principal accomplishments, and his influence on the future of mathematics and science.
7. The Bernoulli family, their relationships with one another and with other mathematicians, their accomplishments and their influence.
8. The life of Gauss, his principal mathematical accomplishments, his predecessors, and his influence on the future of mathematics.



8. The life of Lobachevsky, his principal mathematical accomplishments, his predecessors, and his influence on the future of mathematics.
9. (An appropriately small aspect of) the life of Descartes and his influence on the future of mathematics and science.
10. (An appropriately small aspect of) the life of Fermat and his influence on the future of mathematics and science.
11. The life of Riemann, his principal mathematical accomplishments, his predecessors, and his influence on the future of mathematics and science.
12. The life of Poincare, his principal mathematical AND scientific accomplishments, his predecessors, and his influence on the future of mathematics and science. You could concentrate on his expositions of the philosophy of science and mathematics, if you like.
13. (An appropriately small aspect of) the life of Hilbert, his principal mathematical accomplishments, his predecessors, and his influence on the future of mathematics.
14. The life of Cantor, his principal mathematical accomplishments, his predecessors, and his influence on the future of mathematics.
15. The life of Paul Erdős, his principal mathematical accomplishments, his predecessors, and his influence on the future of mathematics.
16. Women in Mathematics: Sophie Germaine, Sonia Kovalevskaya, Emmy Noether, . . . Gender related difficulties, principal mathematical accomplishments and influences on the future of mathematics.
17. The life of Alexandre Grothendieck, his principal mathematical accomplishments, his predecessors, and his influence on the future of mathematics.
18. The “life” of Nicolas Bourbaki, his principal mathematical accomplishments, his predecessors, and his influence on the future of mathematics.

**Thematic (Be careful if you pick a topic (e.g. 1 and 3 below) which are covered in the class.**

1. Attempts to generalize the quadratic formula to higher degree polynomials. (Cardano, Tartaglia, Galois, Abel, ... )
2. The history of the problem of constructing regular polygons with straight-edge and compass (and other instruments).
3. The history of Euclid's parallel postulate.
4. The evolution of the concept of number, from the Greek view that the only numbers are the whole numbers, through the acceptance of rationals, irrationals, complex numbers.
5. The evolution of modern algebraic notation from the work of al-Khwarizmi through Descartes.
6. The evolution of the concept of function. When did the idea first appear? What did Newton think a function was? Euler? Fourier? Continue through the 19<sup>th</sup> Century.
7. The history of the sphere packing problem first posed by Kepler. Discuss Newton's argument through Gauss' work on lattice packing.
8. Discuss the evolution of the concept of "higher-dimensional spaces" and "vectors" in the work of Argand, Hamilton, Grassmann,...
9. History of Statistics, from its beginning through its development in India and elsewhere in the first half of the 20<sup>th</sup> Century.
10. History of Combinatorics, with a discussion of the place of the work of MacMahon.
11. History of the applications of number theory to cryptography in the 20<sup>th</sup> Century, with a discussion of the current controversy on the export of contemporary cryptographic techniques from the United States.
12. History of fractals and related topics from the early work of Poincare, Julia and others to the Mandelbrot set. The interaction of this subject with computers and computer art.
13. History of the four-color problem from Heawood to its solution in the 20<sup>th</sup> Century.
14. Trace the history of mathematical induction. Be sure to remark on the place of the Peano postulates in this history.

15. Trace the history of Horner's method for the evaluation of polynomials.
16. Trace the history of place value notation.
17. Trace the history of (an appropriate small part of) numerical analysis and approximate calculation.
18. Discuss the origin of the idea of logarithm, discussing Napier and Briggs and subsequent developments.
19. Discuss the development of popular conceptions of higher-dimensional space, starting perhaps with *Flatland*.
20. Discuss some mathematical aspects of cryptography.
21. Discuss the history of the formal theory of computation, including Turing machines, Post systems, recursive functions in general.

**Mathematical emphasis.**

1. History of the traveling salesman problem.
2. Research and write-up the material in the "Sand-Reckoner", giving or reconstructing all computational details. (see Boyer, pp. 123-125.)
3. Same for the "Measurement of the Circles", developing your own complete proofs and giving all details. (see Boyer, pp. 125-126.)
4. Prove Propositions 13 through 17 of Euclid, Book 13. (see Boyer. P. 118.)
5. Describe and give proofs for all the methods given in Boyer for trisecting angles.
6. What should be the rules regarding the proper use of a straightedge with two fixed marks in geometric constructions? (Your results must make it possible to trisect angles by its use, see Boyer, p. 134). How do you use the marked straight-edge to trisect angles? Try to develop a theory of what numbers are "constructible" using compass and marked straight-edge.
7. Discuss the following problem. If you can trisect an angle using geometric tools, can you use the same tools to solve any cubic equation? Some cubic equations?
8. Solve the most difficult Apollonian tangency problem using straight-edge and compass (see Boyer. P. 143)
9. Develop the Law of Cosines along the lines of the geometric proof of the Pythagorean Theorem (see Propositions 12 and 13 of Euclid, Book 2, and Boyer, p. 159.)

10. Prove the Theorem of Menelaus, illustrated in Figure 10.6 of Boyer (p. 164) in three ways: synthetic geometry, trigonometry, and analytic geometry.
11. Prove the two generalizations of Heron's theorem given in the displayed formulas on page 210 of Boyer.

### **Philology**

Philology is the study of original manuscripts or the "biography" of a book. (For example, one of the Charles's stole an old manuscript of Euclid and took it to Paris, where it was lost for a century or so.) Philology can be pretty hard, though. If you attempt this area, read through the extensive notes in Heath's Vol. I of Euclid (Dover edition). The history of Archimedes' *The Method* (the famous palimpsest) is a good possibility.

## **Mathematics 504**

### **History of Mathematics Internet Sources**

Here is a list of useful websites. You can find these yourself and even more by using Google or your favorite search engine.

OSU Math Home Page: [http://library.osu.edu/sites/sel/math/math\\_home.htm](http://library.osu.edu/sites/sel/math/math_home.htm)

OSU History of Math Page: <http://library.osu.edu/sites/sel/math/mathhis.htm>

<http://www-gap.dcs.st-and.ac.uk/~history/> (**Perhaps the most important site – St. Andrews, Scotland**)

<http://convergence.mathdl.org/jsp/index.jsp> (**An online magazine from the MAA covering the uses of history of math in teaching**).

<http://aleph0.clarku.edu/~djoyce/mathhist/mathhist.html> (**Clark University**)

<http://dir.yahoo.com/Science/Mathematics/History/> (**Yahoo**)

<http://www.maths.tcd.ie/pub/HistMath/> (**Trinity College Dublin, Ireland**)

<http://archives.math.utk.edu/topics/history.html> (**University of Tennessee, Knoxville**)

<http://mathforum.org/isaac/mathhist.html> (**Drexel University**)

<http://www.ibiblio.org/expo/vatican.exhibit/exhibit/d-mathematics/Mathematics.html>

<http://www.dcs.warwick.ac.uk/bshm/resources.html> (**British Soc. History of Math**)

<http://www2.sjsu.edu/depts/Museum/aamenu.html> (**San Jose State University**)

[http://www.saxakali.com/COLOR\\_ASP/historymaf.htm](http://www.saxakali.com/COLOR_ASP/historymaf.htm) (**Africa**)

<http://www.mathpages.com/home/ihistory.htm> (**I don't know who this is**)

[http://members.tripod.com/~INDIA\\_RESOURCE/mathematics.htm](http://members.tripod.com/~INDIA_RESOURCE/mathematics.htm) (**India**)

<http://www.dean.usma.edu/math/people/rickey/hm/mini/bib-katz.html> (**A special reading list.**)

<http://www.hpm-americas.org/> **History and Pedagogy of Mathematics**

**If you find any useful sites, please email them to [wyman.1@osu.edu](mailto:wyman.1@osu.edu) for distribution to the class! new**

# MARYMOUNT UNIVERSITY

## COURSE SYLLABUS\*

<b>Course Number</b> MA124-honors	<b>Course Title</b> History of Elementary Mathematics
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<b>Fall Semester</b>	<b>Spring Semester</b> X	<b>Summer Semester</b>	<b>Year</b> 2005
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<b>Name of Instructor</b> Dr. Stephanie Cawthorne
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<b>Meeting Day, Time, and Room Number</b> TF 11:00 – 12:15, Lee Honors Room
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<b>Office Hours:</b> To be announced
<b>Phone Number:</b> <b>Email:</b> stephanie.cawthorne@marymount.edu

### 1. BROAD PURPOSE OF COURSE

The goal of the course is to further develop students' understanding of mathematics using the history of the subject.

2. COURSE OBJECTIVES Upon successful completion of this course, students will be expected to:
- demonstrate familiarity with the historical development of number systems and algebra as well as the use of mathematics in problem solving in other cultures and at other times. Specifically, students will be expected to demonstrate their ability to do calculations and solve problems using historical methods and notations through testing and weekly graded homework assignments. For example, students will be asked to “do long division like the ancient Egyptians, solve quadratic equations like the Babylonians, and study geometry just as the student in Euclid’s day” (from the preface to *The Historical Roots of Elementary Mathematics*).

Throughout the course, students will analyze historical mathematical results and discuss the influences of these results on different cultures. Through creative and interactive discussions with their peers, students will further analyze how various modern mathematical methods would have influenced ancient mathematics.

Each student will also demonstrate a deeper understanding of some aspect of a topic covered through the submission of a written research paper. The paper will either examine the overall mathematics of a particular culture at a given time or will take a mathematical topic and examine the positive and negative aspects of the way various cultures treated that topic over time.

3. TEACHING METHOD (lecture, laboratory, audio-visual, clinical experience, discussion, seminar, tutorial)

The format of this course is lecture/seminar. Active student participation in the discussion of assigned readings is expected. The readings will be from the textbooks and, in addition, will include excerpts from original sources (see suggested reading material).

The in-class portion of the course will consist of both traditional lectures and cooperative group sessions. Students will have to critically analyze and discuss with their peers such questions as, “If you could travel in time, what one mathematical concept would you share with the Babylonians and why? What would be the positive aspects of sharing this knowledge? What would be the negative aspects?”

4. GRADING POLICY (i.e. number of graded assignments, weight given to each)

Course grades will be computed according to the following percentages:

Two hour-tests at 15% each:	30%
Comprehensive final examination:	30%
Group Collaboration	5%
Written paper	15%
Homework	20%

5. TENTATIVE CLASS SCHEDULE (List topics to be covered with *approximate* dates of presentation)

<i>Roots = The Historical Roots of Elementary Mathematics</i> <i>Peacock = The Crest of the Peacock: Non-European Roots of Mathematics</i> Additional readings will be posted on course website		
week of	Tuesday	Friday
<b>11 January</b>	Incan: representation of numbers and information on quipus and counting boards <i>Peacock</i> pp 23-41	Mayan: representation of numbers <i>Peacock</i> pages 41-56 <i>Roots</i> section 8.4
<b>18 January</b>	Egyptian: representation of whole numbers and fractions; arithmetic <i>Roots</i> sections 1-1 to 1-8 <i>Peacock</i> pages 61-76	Egyptian: problem solving; algebra <i>Roots</i> sections 1-9 to 1-11 <i>Peacock</i> pages 76-90
<b>25 January</b>	Finish Egyptian Mathematics/ Begin Babylonian Mathematics	Babylonian: representation of whole numbers and fractions; arithmetic <i>Roots</i> sections 2-1 to 2-5 <i>Peacock</i> pages 96-113
<b>1 February</b>	Babylonian: problem solving; algebra <i>Roots</i> sections 2-6 to 2-9 <i>Peacock</i> pages 113-124	catch-up/review
<b>8 February</b>	<b>Test 1</b>	Greek: representation of whole numbers and fractions; means; figurative numbers <i>Roots</i> sections 3-1 to 3-6

<b>15 February</b>	Greek: Euclidean geometric algebra; the golden ratio <i>Roots</i> sections 3-7 to 3-10, 6-1 to 6-6 geometry; Euclid's <i>Elements</i>	Finish Greek Mathematics/ Begin Roman Mathematics
<b>22 February</b>	<b>Monday Schedule – No Class</b>	Roman: representation of numbers; counting board (abacus) <i>Roots</i> sections 8.1 & 8.2
<b>1 March</b>	Chinese: representation of numbers; oriental abaci <i>Peacock</i> pages 140-148	Chinese: arithmetic; finding roots <i>Peacock</i> pages 156-164
<b>8 March</b>	<b>SPRING RECESS</b>	
<b>15 March</b>	Chinese: algebra <i>Peacock</i> pages 170-177	catch-up/review
<b>22 March</b>	<b>Test 2</b>	<b>EASTER HOLIDAYS</b>
<b>29 March</b>	Indian: representation of numbers; arithmetic <i>Peacock</i> pages 239-249	Indian: problem solving; algebra <i>Peacock</i> pages 258-262 & 272-280
<b>5 April</b>	Islamic: transmission of ideas <i>Roots</i> section 8.3	Islamic: algebra <i>Peacock</i> pages 316-320, 324-328
<b>12 April</b>	Medieval European: representation of fractions; arithmetic	Medieval European: problem solving; algebra
<b>19 April</b>	Medieval European: problem solving; the golden ratio	catch-up/review
<b>25 April</b>	<b>FINALS WEEK</b>	

6. REQUIRED TEXT

Lucas N. H. Bunt, Phillip S. Jones, Jack D. Bedient. *The Historical Roots of Elementary Mathematics*. Dover Publications, 1988.

George Gheverghese Joseph, *The Crest of the Peacock: Non-European Roots of Mathematics*. Princeton University Press, 2000.

7. REQUIRED OR SUGGESTED READINGS OR AUDIO-VISUAL MATERIALS

The following list includes original sources, secondary sources, and an index to web resources. Excerpts from some of the original sources will be posted on the web. Other original and secondary sources will be on reserve in the library.

**Original Sources (with commentary):**

Arnold Buffum Chace. *The Rhind Mathematical Papyrus*. National Council of Teachers of Mathematics, 1979 (Reprint of Mathematical Association of America, 1927-29).

Shen Kangshen, John N. Crossley, and Anthony W.-C. Lun. *The Nine Chapters on the Mathematical Art: Companion and Commentary*. Oxford University Press (Science Press, Beijing), 1999.

O. Neugebauer and A. Sachs, eds. *Mathematical Cuneiform Texts*. American Oriental Society, 1945.

Jacques Sesiana. *Books IV to VII of Diophantus' Arithmetica: In the Arabic Translation Attributed to Qustā Ibn Lūqā*. Springer-Verlag, 1982.



L. E. Sigler. *Leonardo Pisano Fibonacci: The Book of Squares*. Academic Press, 1987.

L. E. Sigler. *Fibonacci's Liber Abaci*. Springer-Verlag, 2002.

Lam Lay Yong and Ang Tian Se. *Fleeting Footsteps: Tracing the Conception of Arithmetic and Algebra in Ancient China*. Including *The Mathematically Classic of Sun Zi*. World Scientific, 1992.

### **Secondary Sources:**

A. Aaboe. *Episodes From the Early History of Mathematics*. Mathematical Association of America, 1964.

Marcia Ascher and Robert Ascher. *Mathematics of the Incas: Code of the Quipu*. Dover Publications, 1997 (Reprint 1981 ed.).

Isabella Bashmakova and Galina Smirnova. *The Beginnings & Evolution of Algebra*. Mathematical Association of America, 2000.

J. L. Berggren. *Episodes in the Mathematics of Medieval Islam*. Springer-Verlag, 1986.

Florian Cajori. *A History of Mathematical Notations*. Dover Publications, 1993 (Reprint of 1929 ed.).

Graham Flegg. *Numbers: Their History and Meaning*. Dover Publications, 2002 (Reprint of 1983 ed.).

Richard J. Gillings. *Mathematics in the Time of the Pharaohs*. Dover Publications, 1982 (Reprint of 1972 ed.).

Sir Thomas Heath. *A History of Greek Mathematics*. 2 vols. Dover Publications, 1981 (Reprint of 1921 ed.).

Karl Menninger. *Number Words and Number Systems: A Cultural History of Numbers*. Dover Publications, 1992 (Reprint of 1969 ed.).

O. Neugebauer. *The Exact Sciences in Antiquity*. Dover Publications, 1969 (Reprint of 1957 ed.).

Donald Smeltzer. *Man and Number*. Dover Publications, 2003 (Reprint of 1974 ed.).

Sherman Stein. *Archimedes What Did He Do Besides Cry Eureka?* Mathematical Association of America, 1999.

B. L. van der Waerden. *Science Awakening I: Egyptian, Babylonian, and Greek Mathematics*. Scholar's Bookshelf, 1988 (Reprint of 1975 ed.).

Index to web resources: School of Mathematics and Statistics, University of St. Andrews. *The MacTutor History of Mathematics Archives*; <http://www-groups.dcs.st-and.ac.uk/~history/>

### 8. MARYMOUNT UNIVERSITY HONOR PLEDGE

As a member of the Marymount University community I agree to uphold the principles of honor set forth by this community, to defend these principles against abuse or misuse, and to abide by the regulations of the University.

### 9. SPECIAL NEEDS AND ACCOMMODATIONS

Please address with the instructor any special problems or needs at the beginning of the semester.

Those seeking accommodations based on disabilities should provide a Faculty Contact Sheet obtained through Disability Support Services (DSS), 284-1615, located on Main Campus in Gerard Hall.

**\*This syllabus is a revised version of a syllabus developed by Judy Green for a mathematics history course that satisfies our general education mathematics requirement.**

**Syllabus for 800:185.01**  
**History of Mathematics: From the Calculus to the 21<sup>st</sup> Century**  
**Spring 2005**

**Description:** Survey of mathematical activities from the development of calculus in the 17<sup>th</sup> century. Rise of analysis, and development of modern algebra, non-Euclidean geometries, and general axiomatic method in the 19<sup>th</sup> century. Set theory, topology, mathematical logic, and other integrating developments in 20<sup>th</sup> century mathematics. This is a fascinating area of study, allowing us to explore most of modern mathematics as it was created. We cannot be exhaustive of this history; rather, we'll be on a guided tour of the material hitting some of the intellectual highlights of mathematics from the past 350 years. The most important part of this course will be the work that you do daily, both in and out of class, on your own and with others. Our text is Victor Katz's *A History of Mathematics: Brief Version*.

**Instructor:** Joel Haack

**Office:** BRC 50    **Phone:** 3-2585    **Home phone:** 266-7862

**Office hours:** By appointment -- Linda Schneider, the CNS administrative assistant, will be happy to arrange a time for you to see me. Because of my administrative responsibility, I may not be able to see you at short notice. So do work on assignments early and ask about them in class!

**e-mail:** [Joel.Haack@uni.edu](mailto:Joel.Haack@uni.edu) (An excellent way to reach me.)

**Aims of the course:**

1. To give life to your knowledge of mathematics.
2. To provide an overview of mathematics -- so you can see how your various courses fit together and to see where they come from.
3. To teach you how to use the library and internet for mathematics.
4. To show you that mathematics is part of our culture.
5. To indicate how you might use the history of mathematics in your future teaching.
6. To improve your reading and analytic skills, especially in a technical situation.
7. To improve your oral and written communication skills in a technical setting.

**Schedule and tests:** Two tests will be given during the course and a third during final exam week. The examinations are not intentionally cumulative. Objective items on each test will cover only material since the previous test, but you may choose to answer essay question(s) that call for a more comprehensive answer. Our tentative time schedule and topics follows, though if your interests indicate that other topics are more appropriate, we'll consider changes:

Weeks 1-2. Introduction to the course, precursors of calculus: Babylonians, Egyptians, Eudoxus/Euclid and Archimedes.

Week 3. Oresme. Napier and Briggs (logarithms)

Week 4. Early 17th Century Tangents and Extrema (Fermat, Descartes, Hudde)

Week 5. Catch-up and first test

Weeks 6-8. Early 17th Century Areas and Volumes, Newton, Leibniz.

Week 9. Eighteenth Century Textbooks and Foundations of Calculus.

Spring Break.

Week 10. Catch-up and second test.

Week 11. Rigor in Analysis of the Nineteenth Century.

Week 12. The Arithmetization of Analysis and Set Theory.

Week 13. Infinitesimals. Algebraic Developments (Abel, Galois, Gauss)

Week 14. Gauss. Computers.

Week 15. Computers. Catch-up.

Final Examination 3:00 Tuesday, May 3.

**Grading** will be based on the following, and on any additional agreements we make during the semester. Each test will count 100 points. For essays on tests, I look for completeness, correctness, and specificity.

Assignments will typically be due on Fridays. Late assignments may receive a deduction. The assignments will cumulatively count 100 points toward the final grade, with points given for legitimate attempts to solve the problems. There may be opportunities to resubmit solutions till they are satisfactory.

The problems in this book contain an important amount of the information of the course. It is important that you at least read all the assigned exercises. Please work together on the problems, if you like, but please do hand in your own work showing your understanding of your group work.

There will also be writing assignments: an automathography, two library/internet assignments, and a (library) research paper. The first several of these are worth 25 points each, while the research paper will be worth 125 in total.

**Automathography:** The first writing assignment involving is to e-mail me an automathography. You are to introduce your mathematical self to me in typical narrative form. Tell me about the courses you have taken, what your favorites were, what you find hardest. Explain what your mathematical interests are, and what you plan to do after graduation. Reveal why you signed up for this course and what you expect to get out of it. If your aims for taking this course are different than those stated above, please let me know. If you have any anxieties about this course, or any special problems or needs, let me know. You are encouraged to be creative in your response; don't be pedantic and just answer the questions asked above; include whatever you wish. There are three purposes for this assignment: to make sure that you are/become comfortable using email to write to me, to introduce yourself to me, and for me to get some idea about your writing skills. Send your response by email by Monday, January 17. (Yes, I realize this is not a class day). The assignment will be graded and worth 25 points.

There will be two **library assignments**; I'll give you details about these shortly.

**Research Paper:** You are to write a paper on a topic of your choice. This is meant to be an interesting and enjoyable assignment, not (at least not merely) a chore. So choose a topic with care.

You should think about the choice of a topic for your paper while you are doing the two library assignments. Some students prefer to write about a mathematician, others prefer the history of some mathematical topic. You are encouraged to talk to me (during office hours, after class, or via e-mail) about possible topics. As soon as you have an idea, please let me know so that I can suggest possible references or make comments about the reasonableness of your choice of topic. By March 3 I would like each of you to give me your topic via email; only rarely will I veto a topic as unreasonable.

Each paper must meet the following requirements:

1. The papers are to be on the history of mathematics. They can be neither all history nor all mathematics. Each should contain a reasonably non-trivial piece of mathematics as well as the history and background of that mathematics.
2. Enough expository material should be included so as to make the paper self-contained. If you have doubts, ask a friend to read it. Having someone else read your paper critically is the best way to improve the exposition.
3. You should use a variety of research materials and must give careful references to your sources. You will want to use textbooks, other books, original sources, journals, web sites, and encyclopedias. Your paper should include a bibliography listing your sources and they should be cited in the body of your paper when appropriate.
4. The paper must be prepared using a word processor (you may write in symbols if the word processor you are using does not handle them). Other issues such as the length, format, etc., are up to you. Since you will be startled by this last comment, let me point out that papers have a natural length.

You are telling a story which needs certain background, exposition, and detail. When that is successfully done, stop; you have finished. You should turn in two copies of your paper as I intend to keep one copy.

The grading of your paper will be based on a number of factors, including: the historical and mathematical content; the significance, interest, accuracy, and completeness of the material; the accuracy, scope and significance of your references, and the sensitivity with which they are used and cited; and finally, the style in which it is written. Poorly written papers will not be accepted. As in Olympic figure skating, your score will be a combination of technical performance and artistic merit. The grade of A will be given only for truly excellent work, most likely using original sources; B for good solid work that makes use of high quality sources; C for average work; D and F for unsatisfactory work. All grades are possible and have been given in the past.

To guarantee that you have time to devote sufficient thought to your paper, a topic is due March 1 and a preliminary one-page report is due March 29. This preliminary report should include (a) your topic, (b) a few words about what you intend to include regarding history and in terms of the mathematics, (c) a few words about what you intend to include of mathematics, (d) a brief outline of how you intend to proceed, (e) your preliminary bibliography in proper form, and (f) any questions you have about your paper. This is best done via email as it makes it easier for me to incorporate comments. The more details you include, the more feedback you will get from me. Feel free to ask questions and to indicate the problems you are having. The intent of this preliminary report is twofold: It should aid you in writing your paper and it allows me to make suggestions. Remember, the secret of good writing is rewriting. This preliminary report and outline is worth 25 of the 125 points.

The final version of your paper is due April 12 or 19. (We'll discuss the dates; whichever one is picked will be a firm deadline.) This paper is worth 100 points.

You may wish to expand on a paper that you have written for another course. If you plan to do this or to use this paper in another course, you must discuss this with me, according to University policy. At a minimum, you will be asked to submit both your original paper and the one written for this course, so that the work you've done for this course can fairly be evaluated.

**Plagiarism:** According to the Random House College Dictionary plagiarism is "the appropriation or imitation of the language, ideas, and thoughts of another author, and representation of them as one's original work." Scrupulous care must be taken to avoid this in your writing. Naturally the source of a direct quotation must be cited. But also when you take the ideas of another and rephrase them you must cite your source. In historical work everything except the common and readily available facts need a reference to the work where you learned this information. Mutilation of library materials is a crime, both literally and figuratively. Photocopying is cheap and readily available, so there is no excuse for defacing library holdings in any way.

Cheating of any form will not be tolerated and will be treated with the utmost severity.